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Cost-Effective, High-Accuracy Inertial Navigation

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ABSTRACT

The trend in today's navigation systems is toward Global Positioning System (GPS)/Inertial Navigation System (INS) hybrids. Whether because of GPS outage, or vulnerability, or the fact that GPS does not supply real-time attitude and heading data, there is a need for a pure, self-contained INS. In addition, certain operational scenarios require high-accuracy position, velocity, attitude, and heading data from an INS. Modern INSs are dominated by ring laser gyro (RLG) technology; these systems generally lie in the 1 nmi/h class. To obtain a factor of improvement demands either development of higher-accuracy (and hence more expensive) inertial sensors, or alternative mechanization of existing inertial sensors.

An alternative mechanization ideally suited to today's RLGs is the rate bias technique. This technique solves many of the problems associated with dithered gyros and also attenuates the effects of many of the inertial sensors' errors.

This paper addresses the system design, the design rationale, and flight test results demonstrating that a rate bias RLG/INS provides a cost-effective solution to the pure, high-accuracy INS requirement.

INTRODUCTION

Four years ago, Litton's Guidance and Control Systems Division initiated the design of a new type of Inertial Navigation System (INS) that can achieve high accuracy—0.1 nmi/h (circular error probability [CEP]) and low velocity error—less than 2 ft/s, 1 σ—for long flight times. The remarkable attribute of this design was that it used inertial sensors that had already been developed for less accurate INSs. A prototype model of this design, designated as the LN-94R, has been flown for over 3 years and has consistently demonstrated navigation accuracy of 0.1 to 0.2 nmi/h (CEP). Figure 1 depicts the rotating sensor assembly; Figure 2 shows a complete INS. This paper describes the design features of this system and the flight test data demonstrating that a cost-effective, high-accuracy INS is now available.

The claims of this paper are such that a very important question must be answered: Why was the rate bias RLG mechanization not developed earlier? There are two principal reasons. First, the rate bias RLG mechanization requires miniaturized electronics if a reliable system is to be achieved. Miniaturized, reliable electronics have only recently been available. Second is fashion. With the advent of strapdown inertial navigation, the fashion has been to design

Fig. 1—Rotating Sensor Assembly

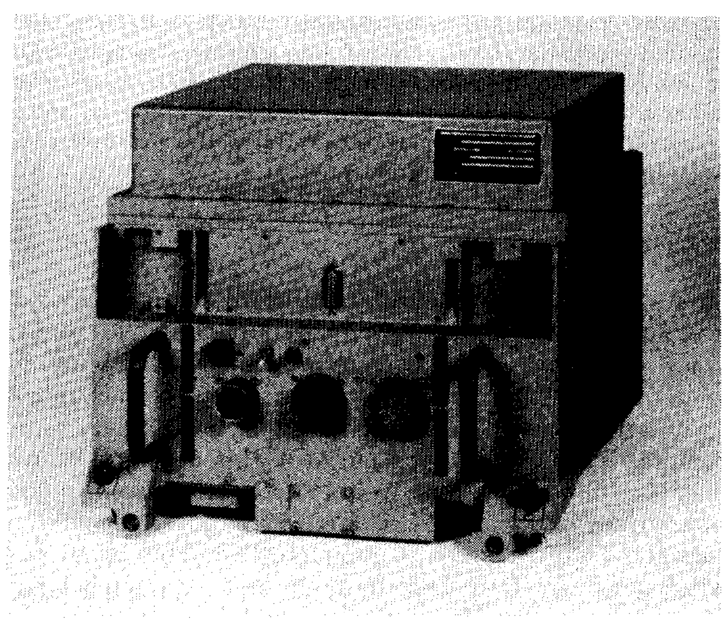

Fig. 2—INS Assembly

solid-state systems (i.e., no moving parts). Conformance with this trend caused many designers to overlook other cost-effective solutions.

The appendix contains some useful equations for INS error analysis.

SYSTEM DESIGN

The inertial sensors used in the LN-94R, three 28 cm ring laser gyros (RLGs) and an A-4 accelerometer triad, are the same type as those being used in commercial INSs (LTN-92) and military aircraft (standard INSs). These sensors nominally perform at slightly less than 1.0 nmi/h (CEP). The performance of the inertial sensors is enhanced by the mechanization implemented, in this case, strapdown rate biasing.

Strapdown rate biasing combines the same algorithms and techniques as those developed for pure strapdown INSs with a steady rotation (rate bias) mechanization of an RLG. The question that must be answered is, Why rate bias an RLG?

Rate Bias versus Dithered Ring Laser Gyro

The majority of RLGs in operation today use a mechanical dither to prevent lock-up of the two counter-rotating light beams when the gyro is subjected to low rates. The disadvantages of dithered RLGs are that the gyro output is contaminated with the dithered rate of approximately 400 Hz, and that random walk, which determines alignment and navigation accuracy, is greatly increased because of the numerous times the gyro goes through zero rate.

In order to obtain high-accuracy navigation performance, a low random walk is required. A simple calculation will demonstrate the first major advantage of rate bias over dither mechanization.

When a dithered RLG goes through zero rate, a small angular error is introduced as the two counter-rotating light beams couple. This angular error is proportional to the time spent in the lock-in rate range, the time being a function of the angular acceleration of the dither at the lock-in rate.

The standard equation for random walk of a dithered RLG is given by [1]:

$$\frac{\sigma_\Omega}{\sqrt{t}} = \frac{fL}{\sqrt{2\pi(SF)fm}} \ \deg/\sqrt{h}$$

where σ_Ω is RMS error in gyro input rotation angle, SF is gyro scale factor, fL is lock-in threshold, fm is peak amplitude dither, and t is data accumulation time.

For each turn-around event, it can be shown:

$$\text{random walk performance} = K\sqrt{\frac{N}{\ddot{\theta}}} \tag{1}$$

where N is number of times in lock-in, $\ddot{\theta}$ is angular acceleration, and K is lock-in related parameter of the gyro.

In the course of 1 h of operation, a 400 Hz dithered gyro will go through lock-in

$$400 \times 2 \times 3600 = 2.88 \times 10^6 \text{ times.}$$

In the course of 1 h of operation, the LN-94R rate bias mechanized RLG will go through lock-in only

$$\frac{50}{720} \times 3600 = 250 \text{ times},$$

for a rate bias scheme of 50 deg/s reversed every 720 deg. Thus the major reason rate bias reduces random walk is simply that it reduces the number of times the laser goes through lock-in. From equation (1) we can compute the ratio of the reduction in random walk if we know the angular acceleration of the two mechanizations.

The angular acceleration of a 400 Hz dithered gyro is 300,000 deg/s^2. The angular acceleration of a reversed rate bias is 3000 deg/s^2.

$$\frac{\text{(random walk performance) dithered}}{\text{(random walk performance) rate bias}} = \sqrt{\frac{(2.88 \times 10^6)(3{,}000)}{(250)(300{,}000)}} = 10.7$$

Thus, a factor of 10:1 improvement is achievable by simply changing the mechanical motion applied to the gyro. However, not all of this improvement is achievable because the spontaneous emission effects associated with RLGs will dominate once lock-in effects have been significantly reduced. Typically, random walk improvements of 5:1 are consistently achieved with rate biased RLGs. The next question to address is, Can this improvement be realized in an economical manner?

Comparison of Dithered and Rate Bias Mechanization Hardware

Two types of dither schemes are possible: individually dithered RLGs and common dithered RLGs. Individually dithered RLGs are the dominant choice in today's RLG INSs.

Table 1 shows a comparison of the hardware required by rate bias, individually dithered, and common dithered gyro mechanizations. A comparison of rate bias and individually dithered mechanizations shows that the former is much simpler and more reliable, and should be lower in cost. A major advantage is that cross-coupling dither effects, a concern in dithered RLG INSs, are completely eliminated. An additional advantage is the elimination of low-frequency isolators that must be used on a dithered RLG INS to maintain correct dither levels and to make the dithered RLG INS immune to mechanical impedances external to the INS.

By eliminating or having very stiff vibration isolation systems, made possible only by the rate bias mechanization, substantial performance improvements in attitude and attitude rates are also realized.

Design of a Rate Bias Mechanized RLG INS

Modern RLG INSs are much more reliable than previous generations of INSs. Obviously, we do not want to lose this advantage when the system mechanization is changed to rate bias. Careful attention must therefore be paid to the rate bias rotation mechanism. A rate bias system requires large angular motion (i.e., a motor), a pickoff to relate the position of the inertial sensors to the INS

Table 1—RLG Lock-In Avoidance Mechanization Hardware Comparison

	Rate Biased	Individually Dithered Gyros	Common Dithered Gyros
Suspension	Slow-speed ball bearing set	Three individual small angle flexures dithering at separate frequencies	One larger flexure suspension with larger angle capability
Drive Mechanism	One dc brushless torque motor	Three piezoelectric driven torsional spring-mass resonant systems	One piezoelectric driven torsional spring-mass resonant system
Electrical	One electrical circuit to drive motor at fixed speed	Three closed-loop amplitude and frequency controlled circuits	One closed-loop amplitude and frequency controlled circuit
Pickoff	One angle pickoff	Three gyro dither pickoffs or built-in optical compensation	One dither pickoff
Gyro Motion Reaction Torques	Negligible	Three-axis torques at separate frequencies, requiring low natural frequency vibration isolation from chassis	Single-axis, single-frequency larger torque, requiring isolation or counterbalance

chassis (hence aircraft datum), control of the inertial sensors on a rotating platform, and one set of bearings.

The choice of a dc brushless motor ensures that no maintenance is required, as would be for brush-type motors.

Both optical encoders and Inductosyn (a device similar to multispeed synchros) have been used successfully as pickoffs on early models of the rate bias INS at Litton, with the preference being toward the former. The accuracy of the angular read-out from the encoder depends on the heading accuracy requirement. A conventional INS usually specifies 3 arcmin (1 σ). This is achieved by choosing a 13 bit encoder, which will give 2.63 arcmin resolution. For certain applications, precision azimuth is required. To read out aircraft heading to 1.2 arcsec resolution, an equivalent 20 bit optical encoder is required. The optical encoder is synchronized with the strapdown attitude reference (quaternions or direction cosines) by means of an electrical index indicator which is part of the optical encoder. This index indicator can also be used to ensure that the rate bias sensor assembly is reversed at the same point. Attitude accuracy during the reversal period (and indeed during any other period) is achieved by synchronizing counters that count the optical encoder pulses and the laser gyro delta theta pulses. Synchronization to better than 1 arcsec has been demonstrated from rate bias INSs.

The control of the inertial sensors on a rotating platform is exactly the same as that employed by dithered INSs. If one examines a dithered INS, it will be seen that over 100 interconnects are made between the inertial sensors and their associated chassis-mounted electronics. If a rate bias INS used the same design rules, then a massive and costly slip ring or an untenably large flexible cable would be required from the inertial sensor to the chassis-mounted electronics. Either of these two design choices would have disastrous consequences. The most efficient way to mechanize the control of the rate bias inertial sensors is one that does the following:

1) Miniaturizes and locates the inertial sensor electronics on the rotating platform.
2) Serializes the inertial data for optical transmission to the stationary chassis electronics (e.g., navigation processor).
3) Generates the various voltages required by the inertial sensors (including the high voltage required by the RLGs) from a single power input.

Using the above design features, it is possible to construct a rotating inertial sensor assembly that requires only two flexible, nonsignal connections. A very simple, low-cost flex cable can be used.

The bearing used in a rate bias system, except in the case where precision attitude and pointing are a requirement, uses medium-precision self-lubricated ball bearings.

Carouselled or Maytag Rate Bias INS

Carouselling is a term normally reserved for a mechanization that turns continuously in one direction, whereas Maytagging means that the direction of rotation is periodically changed. The choice of a Maytag mechanization is preferable because certain inertial sensor parameters can be relaxed. An example of this is the gyro scale factor; if we assume a 1 ppm gyro scale factor error and a rotation rate of 50 deg/s, then for a carousel mechanization, the error growth $= 50 \times 10^{-6} \times 3600 = 0.18$ deg/h. This is unacceptable for a high-performance INS and can be tolerated only if the stability of the gyro scale factor allows for this effect to be calibrated out as gyro bias. To do this, the scale factor stability needs to be on the order of 10^{-3} ppm, which is extremely difficult, if not impossible, to achieve with the two-mode type of RLG.

The Litton Maytag rate bias mechanization, used on the LN-94R, is a 50 deg/s rate with 720 deg of rotation before reversing. The error caused by a 1 ppm gyro scale factor error is $\pm 720 \times 3600 \times 10^{-6} = \pm 2.592$ arcsec over the Maytag period. The error over a longer time period is also effectively bounded to ± 2.592 arcsec. The effect of a ± 2.59 arcsec azimuth error on a 0.1 nmi/h INS is negligible.

From the above it is concluded that the choice is in favor of the Maytag over the carousel mechanization from the point of view of performance. However, other important design parameters must be considered before this choice is made, namely, gyro lock-in error accumulation and sensor block angular acceleration capability.

To reverse direction, the motor must supply the necessary torque to get the RLG through the lock-in zone quickly. The carousel mechanization obviously has none of these errors since the sensor block is never reversed. If the moment of inertia is optimal and the correct torque motor is selected, then these errors can be made acceptable in the Maytag mechanization.

Reversal of Rate Bias System

The method currently used in reversing the rotation of the LN-94R rate bias INS is solely by means of a torque motor. This is not the most efficient method because large peak currents from 3 to 6 A are required. These current spikes can be eliminated if a spring element is introduced into the reversal mechanism. The spring element conserves the kinetic energy of rotation, and also controls the angular acceleration and deceleration of the inertial sensors. The dc brushless motor size can be reduced because now it is required only to make up for the frictional losses. If the concept of gyros crashing into a spring restraint seems extreme, a simple calculation of the forces applied to the RLGs will show how this compares with a normal dithered mechanization.

Dithered RLG Applied Forces

The parameters for a dithered RLG are $\ddot{\theta}$ = 300,000 deg/s^2 (peak), $\dot{\theta}$ = 120 deg/s (peak), and R (radius to mirrors, detectors, seals, etc.) = 2 inches for the 28 cm RLG.

The centripetal acceleration is given by

$$\dot{\theta}^2 R = (120/57.3)^2(2/12)(1/32.2) = 0.02g$$

Also, tangential acceleration is given by

$$\ddot{\theta} R = (300{,}000/57.3)(2/12)(1/32.2) = 27.1 \text{ g}$$

A 27.1 g, 400 Hz vibration is a severe environment for any device.

Rate Bias RLG Applied Forces

For a rate bias RLG, the parameters are $\ddot{\theta}$ = 3000 deg/s^2 (peak), $\dot{\theta}$ = 50 deg/s (steady), and R (radius to mirrors, detectors, seals, etc.) = 2 inches for the 28 cm RLG.

$$\dot{\theta}^2 R = (50/57.3)^2(2/12)(1/32.2) = 0.003 \text{ g}$$

Also

$$\ddot{\theta} R = (3000/57.3)(2/12)(1/32.2) = 0.27 \text{ g}$$

Comparing the rate bias forces with the corresponding dithered induced forces shows that the applied g level has been reduced by a factor of 100, and the frequency (of the applied force) reduced by 11,520 ($2.88 \times 10^6/250$). Hence, failure caused by fatigue factors is greatly reduced by implementation of the rate bias mechanization.

IMPROVEMENT IN NAVIGATION PERFORMANCE DUE TO RATE BIAS

Rate bias requires that the RLG inertial sensors be rotated through a large angle. Once we have provided the mechanism to accomplish this, many other benefits materialize; namely, certain gyro and accelerometer errors become bounded, which results in improved navigation and alignment performance.

Performance improvement via large-angle rotation has been successfully demonstrated by the Delco Carousel INS for many years. Carouselling reduces system error by averaging the inertial sensor errors. Rate biasing also reduces error by averaging in addition to improving random walk.

An example of the improvement of navigation performance due to rate biasing is shown by the short-term response to level accelerometer bias error.

For a conventional Schuler-tuned INS, the velocity error, v(t), due to acceleration bias, a_b, is given by

$$v(t) = \frac{a_b}{\omega_S} \sin \omega_S t$$

where ω_S is the Schuler frequency. When t < 200 s, $\sin \omega_S t \approx \omega_S t$, and $v(t) \approx a_b t$.

At t = 200 s, v = 200 a_b ft/s. The mean velocity error over the 200 s period is

$$v(t)_{mean} = 100\ a_b \text{ ft/s.} \tag{2}$$

For the same INS rate biased

$$v(t) = \frac{a_b}{\omega_{RB}} \cdot \sin \omega_{RB} t$$

where ω_{RB} is rate bias rotation rate (which is approximately 50 deg/s) for the LN-94R.

The peak velocity error for the rate bias INS will be

$$v_{pk} = \frac{a_b}{\omega_{RB}} = \frac{57.3}{50} a_b \ldots . = 1{\cdot}14\ a_b \text{ ft/s} \tag{3}$$

By comparing the results from equations (2) and (3), it will be seen that the velocity error caused by an accelerometer bias error is greatly reduced by rate biasing. This simple analysis demonstrates the unique attribute of the rate bias mechanization. This mechanization was introduced to improve RLG performance. With no additional hardware, it makes possible the attainment of a high-accuracy accelerometer requirement, which was the other major obstacle to achieving high-accuracy inertial navigation.

Summarizing the unique characteristics of a rate bias RLG INS, we see the following:

1) Gyro performance is improved.
2) Accelerometer performance is improved.
3) Self-generated forces on the inertial sensors are reduced.
4) Low-frequency isolators can be eliminated, leading to improved attitude and attitude rate performance.
5) The built-in-test (BIT) is greatly improved; i.e., the rate bias rotation is sensed by all gyros, and hence the dynamic range of the BIT can be greatly extended.
6) The level accelerometers can be subjected to $\omega^2 R$ and $\ddot{\theta} R$ acceleration change; thus, the accelerometer and its sensitivity are fully tested.

Although the rate bias RLG INS offers many improvements, one should remember that it is generally more difficult to calibrate a rate bias RLG INS than a dithered RLG INS. This is because the gyros are in constant motion at a high rate and are therefore not individually directly observable. The concept of virtual gyros (i.e., body fixed equivalent gyros) greatly improves the human ability to comprehend the calibration process. Calibration of the accelerometer tends to be easier and less costly since one can use the rotation positioning mechanism implicit in the rate bias mechanization.

To determine how well a rate bias RLG INS was calibrated, special flight testing was performed, which, together with other flight testing, is described in the following section.

FLIGHT TESTING OF THE LN-94R RATE BIAS RLG INS

Over the past 3 years, the LN-94R has been subjected to many flight tests, either by itself or with other INSs. Testing of the LN-94R included the following: a Litton flight test on a Cessna II Citation Jet, road (land navigation) tests in a Chevrolet Blazer truck [2], a flight test by McDonnell Douglas as part of the SRAM II competition, and a flight test by the Royal Air Force (RAF) of Great Britain as part of its NIMROD navigation requirement evaluation.

Company Flight Tests

The Litton flight tests were conducted to assess performance capabilities, verify calibration techniques, and prepare the system for the RAF tests. These tests were conducted in Southern California from the Van Nuys Airport. After very few tests had been conducted and after the resolution of the navigation processor software had been improved, 0.1 to 0.2 nmi/h (CEP) performance was consistently achieved. The velocity accuracy of the system was difficult to measure because of significant gravity anomalies that are experienced in the Southern California area. To ascertain the velocity accuracy, it was necessary to apply gravity anomaly corrections (post-flight data reduction) and move to a test area where the effect of the gravity anomalies was much reduced. Accordingly, some of the flight tests were conducted out of Roswell, New Mexico.

Verification of Calibration Techniques

If not correctly calibrated, a strapdown INS will generate large velocity errors when subjected to dynamic maneuvers. A particularly severe test for a strapdown INS is described below.

The "Achilles Heel" of all strapdown INSs is exposed when an aircraft undergoes "S" turns. This type of maneuver causes tilts that produce velocity error along the flight path due to gyro scale factor, and tilts that produce velocity error perpendicular to the flight path due to gyro x-y axis nonorthogonality. A simple way of analyzing this phenomenon is to consider that all coordinated aircraft turns cause positive rotation about the aircraft pitch axis. Therefore, there is always an ongoing accumulation of positive angle, except during the short term of pitching nose down when descending in altitude without turning. This phenomenon is most devastating when the aircraft moves with a serpentine motion continuously along a given flight path.

For example, consider the effect of an aircraft moving with a serpentine motion of ± 45 deg bank and ± 45 deg heading change along a given flight path for 10 such cycles. The total rectified heading change (ΔH) is 10 × 180 = 1800 deg. The accumulated pitch change (ΔP) is given by ΔP = ΔHsin (bank angle) = 1800 sin (45 deg) = 1273 deg.

For a 1 ppm gyro scale factor error, this will produce a tilt error of 4.6 arcsec about the pitch axis. This tilt error would then induce a sinusoidal velocity error of 0.572 ft/s.

This phenomenon was used to evaluate the gyro scale factor and gyro misalignment errors of the LN-94R during flight testing at Roswell, New Mexico in a Citation II Jet. A special flight was conducted to determine these particular system errors. Figure 3 illustrates the results of this flight, which include a north-south velocity error of 1.9 ft/s (peak-to-peak) and an east-west velocity error of 2.5 ft/s (peak-to-peak).

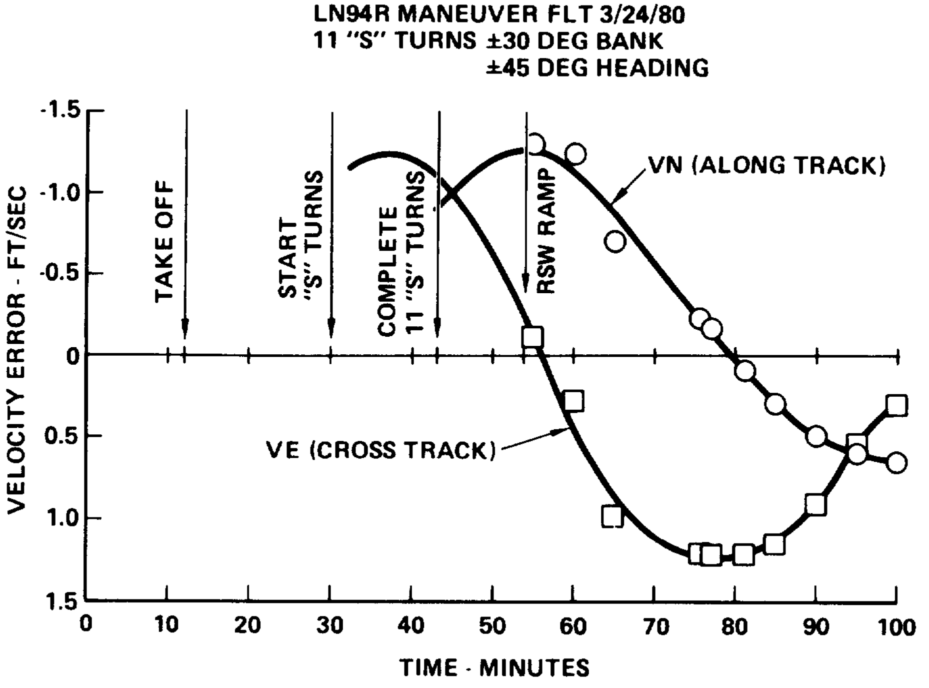

Fig. 3—Post-Flight Schuler Oscillation after Serpentine Maneuver Flight Path

This translates into a north-south tilt (along flight path) of 37.4 μrad and an east-west tilt (cross track) of 49 μrad. If all of this tilt is caused by only the gyro scale factor and gyro x-y misalignment, the system calibration errors can be calculated as follows:

ΔH change	= 11 × 180
	= 1980 deg
ΔP change	= 1980 sin (30 deg)
	= 990 deg
	= 17.3 rad
Gyro SF error	= 37.4 μrad/17.3 rad
	= 2.2 ppm

X-Y gyro misalignment = 49 μrad/17.3 rad
= 2.8 μrad
= 0.56 arcsec

These small numbers demonstrate that the technique used to calibrate the LN-94R is sufficiently accurate for a high-accuracy INS.

Effect of Gravity Anomalies

As stated previously, during flight tests in the company Citation II aircraft, it was soon discovered that the accuracy capability of the LN-94R with regard to velocity could not be adequately demonstrated or realized while flying in the California area because of substantial gravity anomalies in the region of the Sierra Mountains, and also because of extensive fault systems. When flying north-south over the Sierras between Van Nuys and Lake Tahoe, 5 ft/s peaks were typically generated, although general navigation accuracy was on the order of 0.2 nmi/h.

To determine the velocity accuracy of the LN-94R, the base of operations was moved from Van Nuys, California to Roswell, New Mexico, with a flight plan over the Plains area in order to fly in a location with reduced variation in the deflection of the local vertical along the flight path. The results of this plan proved fruitful, as can be seen by the data plotted in Figure 4. This figure shows that the radial velocity peak error $[(Vn^2 + Ve^2)^{1/2}]$ at the end of the flight was 2.1 ft/s. The position accuracy is approximately 0.17 nmi/h. The position data

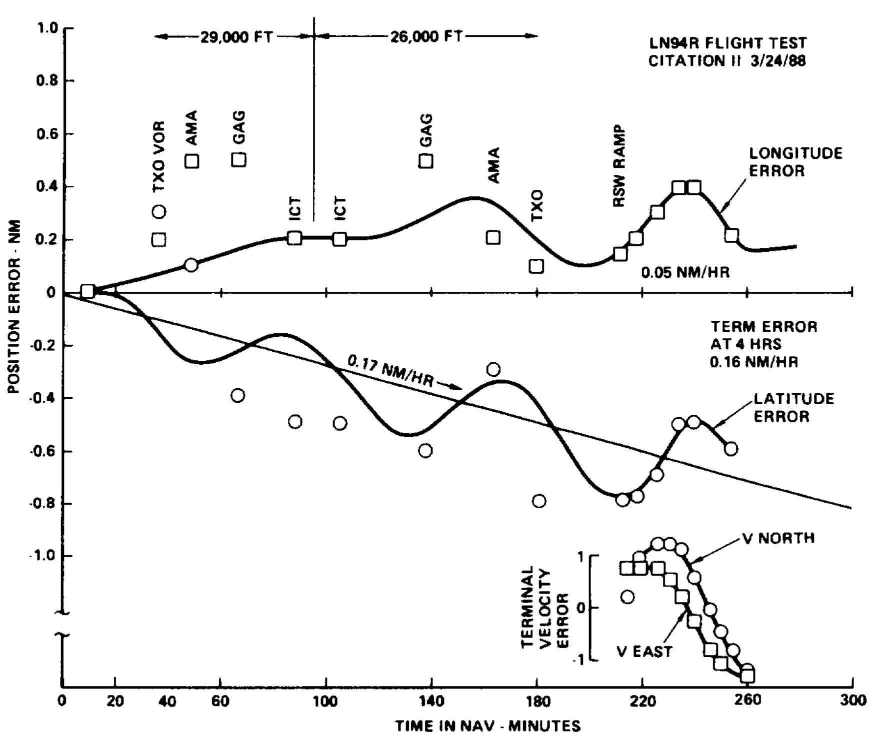

Fig. 4—LN-94R Flight Conducted at Roswell, New Mexico

was ascertained by overflying VOR stations (and performing on-top position measurements), and also measuring the position drift error at the end of the flight. It should be noted that the on-top VOR position measurements were conducted with the aircraft at 29,000 and 26,000 ft. It is estimated that the position error associated with this method of measurement is approximately 0.3 nmi.

LN-94R Flight Test at Boscombe Down, England (1988)

During August 1988, the RAF of Great Britain conducted a series of tests of various INSs, including the LN-94R, on a Comet 4C test aircraft.

These flights were designed to determine the ability of a pure INS to meet high accuracy requirements. A plot of latitude versus longitude of a typical flight is shown in Figure 5 and a plot of heading changes in Figure 6. To determine the velocity accuracy, the aircraft landed after approximately 4.5 h of flying and stayed on the ground for 90 min to observe the Schuler peaks. Then with the system still in the navigate mode (no realignment), the aircraft took off again, repeated the previous scenario of maneuvering, and landed at approximately 10.5 h of total navigation time. Again the system was allowed to Schuler for 90 min. Five of these two-stage flights were flown, for a total of 10 takeoffs, maneuver flights, and landings. The overall composite of these flights is shown in Figure 7.

The CEP of all flights shown is less than 0.1 nmi/h. It should be noted that this error includes the contribution due to gravity anomalies.

The RMS velocity at 8 h was approximately 2.0 ft/s after correction for gravity anomaly effects.

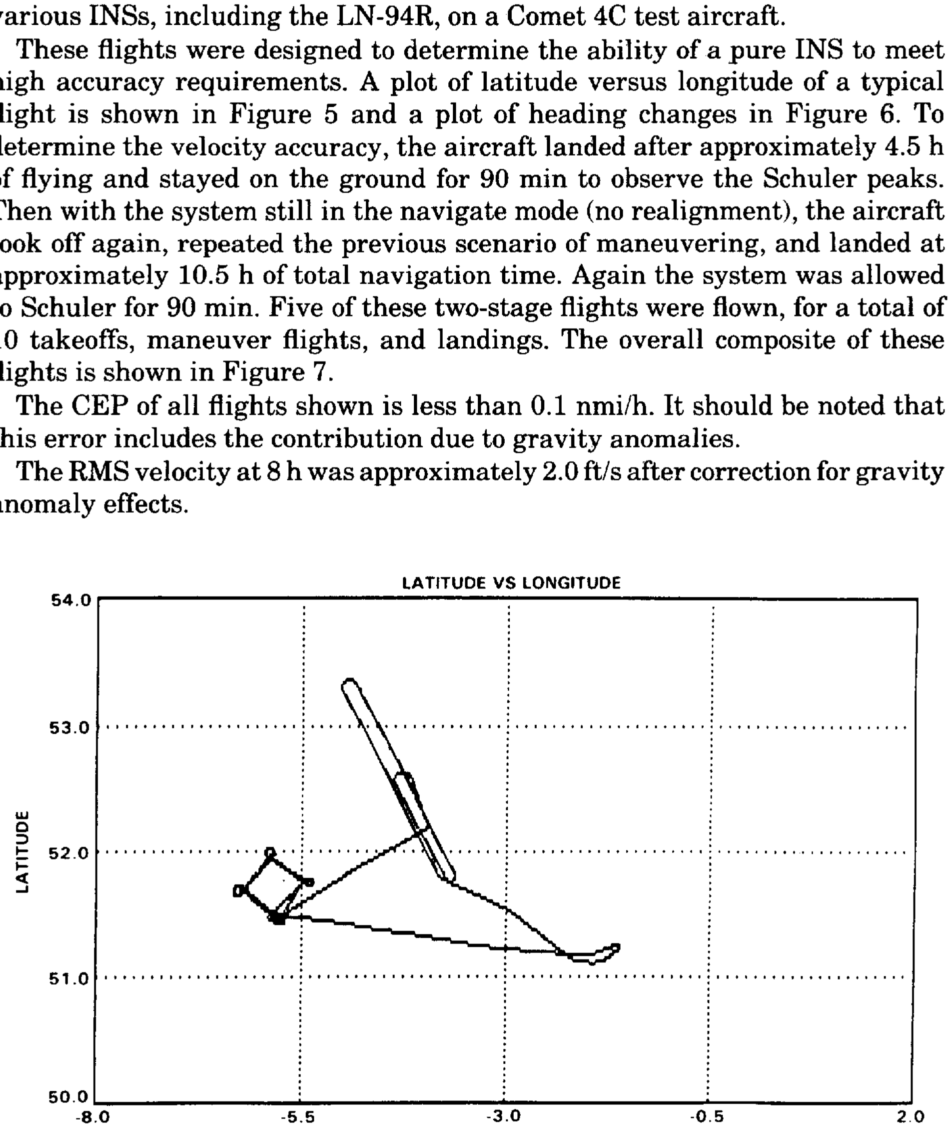

Fig. 5—Typical Antisubmarine Warfare Profile

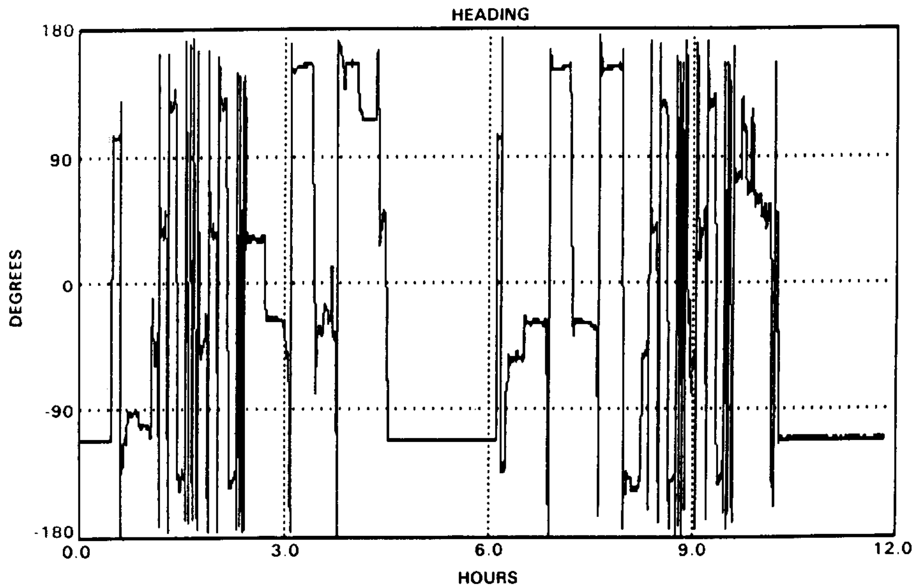

Fig. 6—Heading Changes

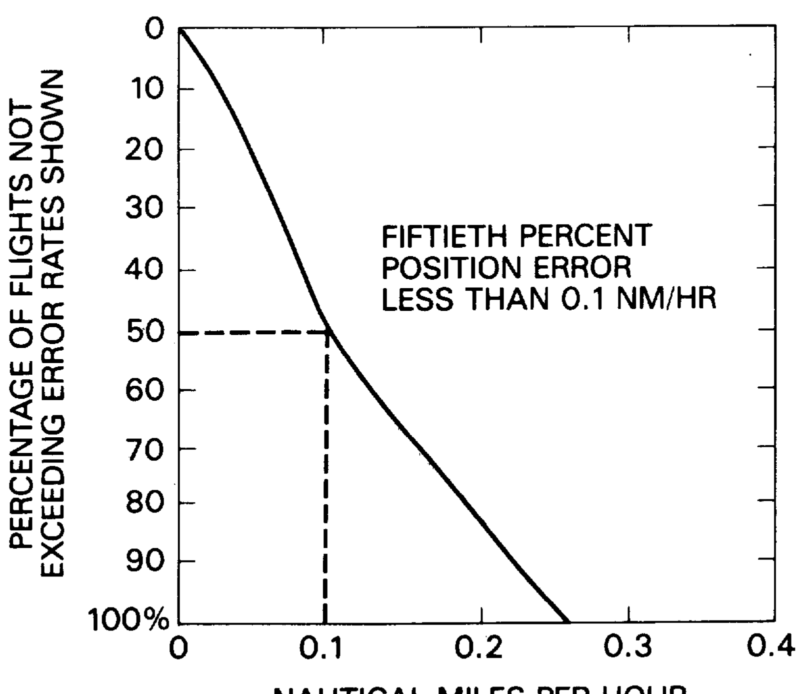

Fig. 7—Boscombe Down CEP Data

RAPID ALIGNMENT CAPABILITY

In addition to having high-accuracy capability, the LN-94R system also has the capability to achieve its performance with only a 5 min alignment time. Theoretically this is possible, since inertial sensor "turn-on" transients, the principal inhibitors of rapid alignment capability, are averaged out by the wide-angle oscillating rotation. This theory is proven out in reality in the LN-94R, as can be seen from the rapid alignment tests described below.

To achieve 0.1 nmi/h, an INS must be aligned to the earth reference (through a process known as gyrocompassing) to an accuracy of 0.008 deg in heading, this being referred to as the alpha (heading) error. With this alpha error and an assumed aircraft velocity of 600 kn, a cross-track position error of $\left(600 \times \frac{0.008}{57.3}\right) \approx 0.1$ nmi/h would result.

From Figure 8 it will be seen that the alpha error at 5 min is less than 0.004 deg, thus showing a performance capability of 0.05 nmi/h.

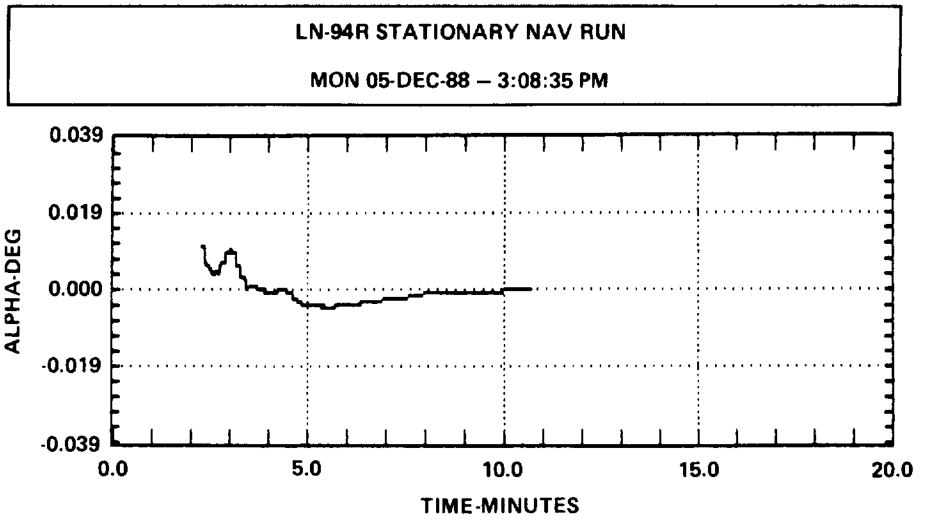

Fig. 8—LN-94R Cold Start Alignment Showing Wander Angle (Alpha) During Gyrocompassing

CONCLUSION

Flight tests conducted over the last 3 years have demonstrated that a rate bias RLG INS (as embodied in the LN-94R) is capable of providing high-accuracy pure inertial navigation performance at a cost that is competitive with today's medium-accuracy INSs. It has been shown that by the addition of a simple large-angle rotation mechanism, many of the problems encountered by dithered RLG INSs are eliminated.

Systems similar in design to the LN-94R are capable of providing precision position, velocity, attitude, and pointing data for aircraft, marine, and land navigation/surveying applications.

Based on a paper presented at The Institute of Navigation National Technical Meeting, San Mateo, CA, January 1989.

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APPENDIX

NAVIGATION ERROR EQUATIONS

The standard error equations that are used in INS error analysis are shown in block diagram form in Figure A1. This block diagram shows two orthogonal level axes (x and y) in which the x axis is initially aligned to true north. The errors in the level axis propagate with a frequency of approximately 84.4 min (Schuler frequency). The third axis of an INS is the azimuth axis.

The 24 h period oscillations associated with INSs that operate for long time periods are caused by the error coupling of the two-level navigation axis via the following terms:

1) Latitude error
2) East velocity error
3) Azimuth rotation errors

The analytic solutions to these equations are tedious, but with today's com-

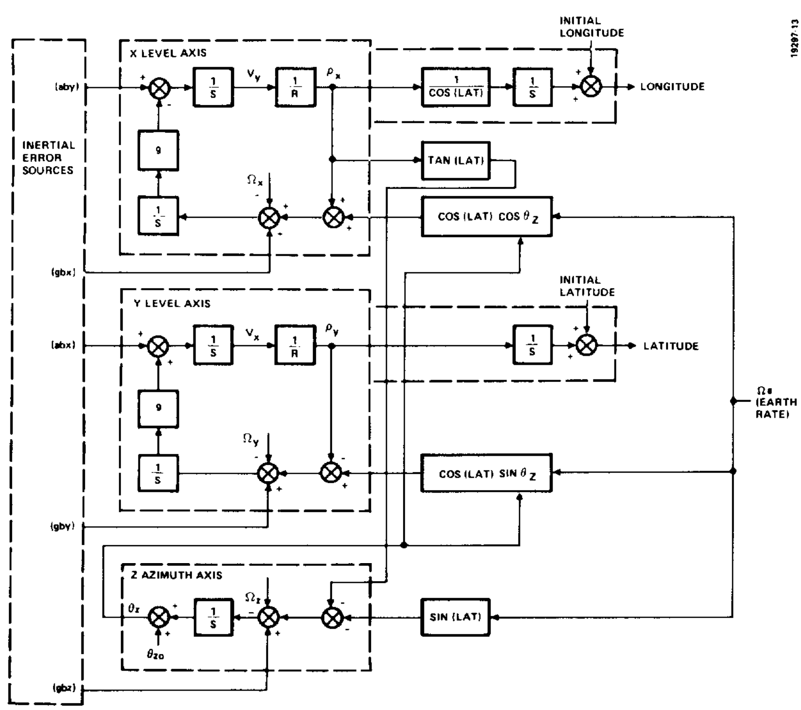

Fig. A1—Inertial Sensor Error Equation Block Diagram puter processing capability, the effect of any error source (e.g., gyro bias, accelerometer bias) can be easily ascertained. However, most practitioners of the art of INSs prefer to operate with a simplified set of error equations. These simplified equations are based on the time period over which the INS or Inertial Guidance System functions:

1) A missile guidance system operates for seconds to minutes; thus Schuler and 24 h effects can be ignored.
2) A fighter aircraft mission is from a few minutes to 2 h; thus Schuler effects are important, but 24 h effects can be ignored.
3) A transport aircraft, ASW aircraft, or ship's navigation system is employed over several hours or even days; therefore Schuler and 24 h effects are very important.

Accordingly, the following set of error equations for velocity and position in the x or y channels is useful for medium-term missions when $Tm < 3$ h:

$$v(t) = \frac{a_b}{\omega_S} \sin \omega_S t + g_b (1 - \cos \omega_S t) \qquad \text{(A1)}$$

$$p(t) = \frac{a_b}{\omega_S^2}(1 - \cos \omega_S t) + g_b R \left(t - \frac{1}{\omega_S} \sin \omega_S t\right) \qquad \text{(A2)}$$

where R is earth radius, ω_S is Schuler frequency, g_b is gyro bias error, and a_b is accelerometer bias error.