The Design of an Effective Marine Inertial Navigation System Scheme

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Abstract

Accuracy of Marine Inertial Navigation System(INS) is mainly dominated by gyroscope. In this paper, a new scheme (Inertial Gyrocompass System) is designed to realize high accuracy INS on the basis of low accuracy float gyros and velocity information of Doppler log. By choosing optimal 2-order horizontal damping network and 1-order azimuth damping network and designing logical damping parameter, the positioning error caused by gyro drift can be eliminated to a large extent. Computer simulation and experimentations show this scheme is effective and practicable.

1. Introduction

INS is widely used in many military and civil scopes because of its ability of independent operation and high attitude, velocity and position information. The primary disadvantage of INS is that its position error increases with the time[6]. From error analysis of INS, we can find that gyro drift is the main error source. That is to say, gyro performance dominates the accuracy of the whole system.

There are two methods to improve the accuracy of INS. One method is to develop new inertial sensor and the other method is by using system technology to improve the accuracy of INS. By logical choice and design of horizontal damping network, azimuth damping network and external velocity error compensation in addition, inertial gyrocompass system can be realized to eliminate position error to a large extent.

2. Design of Horizontal Damping Network

2.1 Analysis of INS error

The most state of INS is no-damping state, because INS is designed to work in Schuler condition. In this state, vehicle acceleration cannot influences on the system[2]. But there are three kinds of periodic oscillation in this state: 84.4 minutes Schuler oscillation, 24 hours earth oscillation and Foucault oscillation that modulates Schuler oscillation and changes with Latitude. For marine system which always works in a long time continuous state, these oscillations must be damped to improve accuracy of INS. HDN (horizontal damping network) can damp Schuler oscillation, meanwhile the Foucault oscillation disappeared.

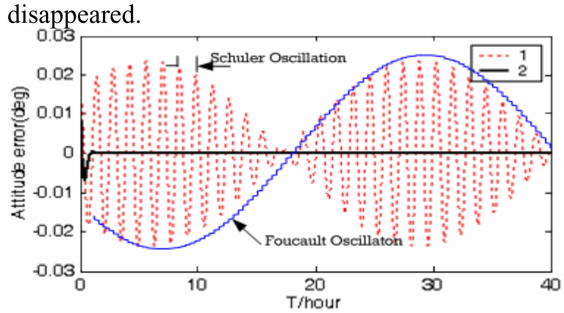

Fig.1 Attitude error curve of INS

In Fig.1, curve 1 is northern-horizontal angle error curve of no-damping state, curve 2 is northern-horizontal angle error of horizontal damping state which damping coefficient (ξ) is 0.5. From Fig.1, we can find Schuler and Foucault oscillation are all disappeared.

Constant gyro drift can be compensated accurately by using gyro measurement, but random drift can not be compensated. In no-damping state, random drift adds the system as a series of impulse, its influences on the system accumulatively. The result is that system error increases with the time.

In order to know the influence caused by random drift, a linear oscillator which input and output relationship is equal to relations of gyro and INS errors (velocity and attitude errors on horizontal axis) is given to describe the influence[3].

2-order damping linear oscillator transfer function is:

$$F(s) = \frac{\omega_0}{s^2 + 2\xi\omega_0 s + \omega_0^2} \tag{1}$$

Where ξ is damping coefficient.

If white noise is added to $F(s)$ as input, mean square root can be expressed as equation (2) and (3).

$$\overline{e_0^2(t)} = \frac{\pi P}{2\omega_0}\left(\omega_0 t - \frac{1}{2}\sin 2\omega_0 t\right) \quad \xi = 0 \tag{2}$$

$$\overline{e_0^2(t)} = \frac{\pi P}{4\omega_0\sqrt{1-\xi^2}}\Big[\left(n+\frac{1}{n}\right)\left(1-e^{-2\xi\omega_0 t}\right) + e^{-2\xi\omega_0 t} n \cos 2\sqrt{1-\xi^2}\,\omega_0 t - e^{-2\xi\omega_0 t}\sin 2\sqrt{1-\xi^2}\,\omega_0 t - n\Big] \quad 0 < \xi < 1 \tag{3}$$

Where P is average power per frequency unit.

In order to make the expression visually, output of $\sqrt{2\omega_0 \overline{e_0^2} / \pi P}$ is given as Fig.2.

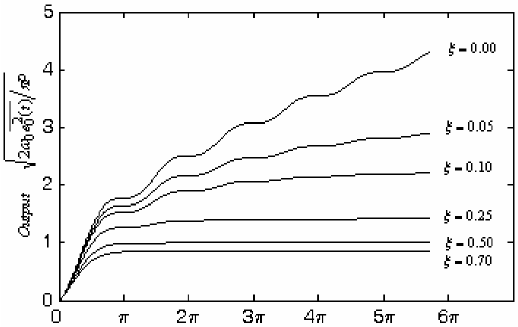

Fig.2 Curve of damping linear oscillator

From result of Fig.2, we can find that the larger damping coefficient is, the smaller mean square root is.

2.2 Design of horizontal damping network

The HDN can be realized in three kinds of networks, they are 1-order network, 2-order network and 3-order network. Considering network and performance and realization difficulty, is the best choice to damp Schuler oscillation[1].

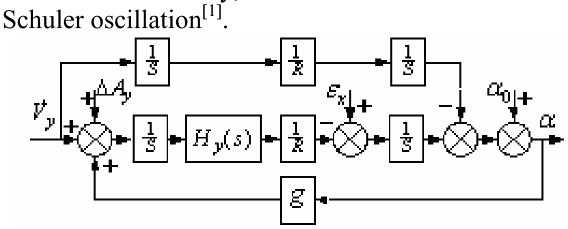

Fig.3 Block chart of signal northern-horizontal axis of INS

Here R is earth radius; ε_x is gyro drift of eastern axis; ΔA_y is original accelerator error; $H_y(s)$ is horizontal damping network.

Fig.3 gives signal flow chart of INS so as to make analysis simply. We should take the following principle into account during the course of choice:

1) The first principle is that the HDN ought to damp Schuler oscillation with angular frequency ω_s =1.24×10^{-3}(rad/s).
2) During the choice, we should also know that larger the damping coefficient is, the more influence will be on the system by vehicle. Thus the coefficient is less than 0.5.
3) The third principle is that HDN must approach one in order to make platform coordinate follow geographic coordinate accurately.
4) The final principle is that HDN must make the system stable. That is to say, HDN should provide positive phase at Schuler angular frequency ω_s .

According to Fig.3, output of $\alpha(s)$ can be described by

$$\alpha(s) = F(s)\varepsilon_x(s)\frac{R}{gs} + F'(s)\frac{\dot{V}_y(s)}{R} \tag{4}$$

Where $F(s) = \frac{G(s)}{1+H_y(s)G(s)}$; $G(s) = \frac{g}{R}\frac{1}{s^2}$

$$F'(s) = \frac{G(s)}{1+H_y(s)G(s)}[1-H_y(s)] .$$

From equation (4), we can find that $F(s)$ is not equal to the closed loop transform function in control system, it is multiplied by $1/H_y(s)$. So try method is the unique method to design damping network.

First, comprising between characteristic of HDN and difficulty of realization, 2-order damping network is the fist choice, function (5) is a good equation which can meet principle 3).

$$H_y(s) = \frac{s^2 + \omega_0 s / p + \omega_0^2}{s^2 + \omega_0 s / q + \omega_0^2} = \frac{(s+\omega_5)(s+\omega_8)}{(s+\omega_6)(s+\omega_7)} \tag{5}$$

Using equation (5), $F(s)$ can be described by:

$$F(s) = \frac{\omega_s^2 s^2 + \omega_s^2 \omega_0 s / q + \omega_s^2 \omega_0^2}{s^4 + \omega_0 s^3 / q + (\omega_s^2 + \omega_0^2)s^2 + \omega_s^2 \omega_0 s / p + \omega_s^2 \omega_0^2} \tag{6}$$

In this paper, $\xi = 0.5$, $\xi = 0.3$, $\xi = 0.1$ are chosen. In higher than 2-order system, its damping coefficient is equivalent damping coefficient. For example, if equivalent damping coefficient is $\xi = 0.5$, the system closed loop gain should have 1.239dB peak value.

There are magnitude and phase of equation (6):

$$M(H) = |H(s)| = \sqrt{\frac{(\omega^2 - \omega_0^2)^2 + \omega^2\omega_0^2 / p^2}{(\omega^2 - \omega_0^2)^2 + \omega^2\omega_0^2 / q^2}} \tag{7}$$

$$\varphi(H) = arctg\frac{\omega_0\omega / p}{(\omega_0^2 - \omega^2)^2} - arctg\frac{\omega_0\omega / q}{(\omega_0^2 - \omega^2)^2} \tag{8}$$

And equation (7), (8) are drawn from equation (5)

$$\omega_5 = -\omega_0(1+\sqrt{1-4p^2})/2p \, , \omega_8 = -\omega_0(1-\sqrt{1-4p^2})/2p \tag{9}$$

$$\omega_6 = -\omega_0(1+\sqrt{1-4q^2})/2q \, , \omega_7 = -\omega_0(1-\sqrt{1-4q^2})/2q \tag{10}$$

From equation (9) and (10), it can be seen that at very low and very high frequency

$M(H) \approx 1, \varphi(H) \approx 0$; and at frequency ω_0, $\varphi(H) = 0$ and $M(H)$ arrives at maximum value q/p.

According to the final choice principle that the system must be stabilized, it is expected that $H_y(s)$ must be positive phase at Schuler frequency ω_s so ω_0 must be bigger than ω_s, this means $p < q$. Meanwhile, in order to assure $H_y(s)$ has real root, p and q must be less than 0.5.

Try three parameters p, q and ω_0 gradually, and verify characteristic of equation (6) according to the second choice principle of HDN, the three parameters p, q and ω_0 are determined :

$$p = 0.1410,\ q = 0.4150,\ \omega_0 = 6.2 \times 10^{-3}$$

Then

$$\omega_5 = 4.308 \times 10^{-3},\ \omega_8 = 8.923 \times 10^{-4},$$
$$\omega_6 = 1.164 \times 10^{-2},\ \omega_7 = 3.303 \times 10^{-3}$$

2.3 Horizontal damping network simulation

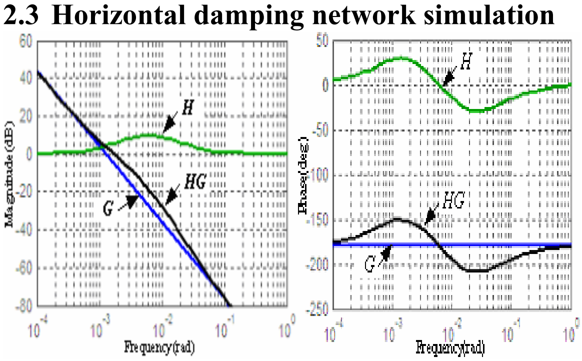

Fig.4 Magnitude and Phase characteristic of $H_y(s)$

From Fig.4, we can find $H_y(s)$ can satisfy principle 1) to 4). In addition, there have a very perfect 3-order HDN[5] $H'(s)$ with $\xi = 0.5$, it can be used to exam characteristic of $H_y(s)$

$$H'(s) = \frac{(s + 8.8 \times 10^{-4})(s + 1.97 \times 10^{-2})^2}{(s + 4.41 \times 10^{-4})(s + 8.8 \times 10^{-3})^2}$$

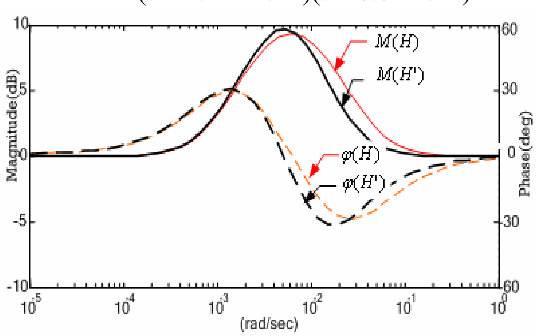

Fig.5 Bode diagram of $H_y(s)$ and $H'(s)$

The magnitude and phase diagrams of $H_y(s)$ and 3-order HDN $H'(s)$ with both $\xi = 0.5$ are shown in Fig,5.

The magnitude and phase diagrams of $F(s)$ and $F'(s)$ that $H'(s)$ is introduced into $G/(1+GH')$ are shown in Fig.6.

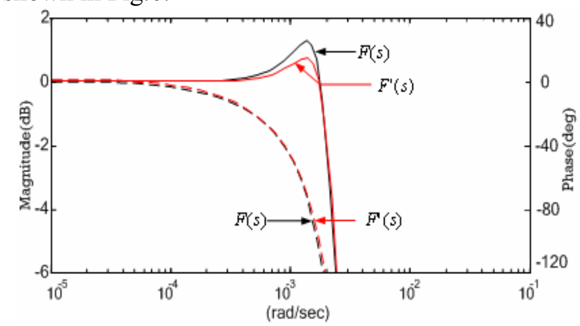

Fig.6 Bode diagram of $F(s)$ and $F'(s)$

From Fig.5 and Fig.6, it can be find that $H_y(s)$ and $H'(s)$ have exactly the same characteristic, so it is verified that HDN $H_y(s)$ is optimal.

2.4 Low damping coefficient of HDN

Using the same method, HDN parameter with $\xi = 0.3$, $\xi = 0.1$ can also be chosen, the result is:

$H_{1y}(s)$ ($\xi = 0.3$):

$$\omega_5 = 3.650 \times 10^{-2},\ \omega_8 = 1.157 \times 10^{-3},$$
$$\omega_6 = 1.273 \times 10^{-2},\ \omega_7 = 3.319 \times 10^{-3}$$

$H_{2y}(s)$ ($\xi = 0.1$):

$$\omega_5 = 2.668 \times 10^{-2},\ \omega_8 = 1.584 \times 10^{-3},$$
$$\omega_6 = 1.752 \times 10^{-2},\ \omega_7 = 2.450 \times 10^{-3}$$

The magnitude and phase diagrams of $H_{1y}(s)$ and $H_{2y}(s)$ are shown in Fig.7.

The magnitude and phase diagrams of $F_{1y}(s)$ and $F_{2y}(s)$ are shown in Fig.8.

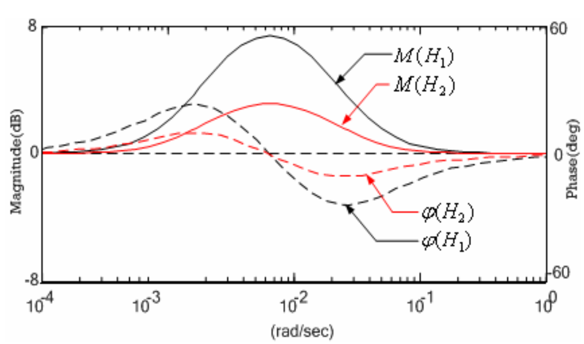

Fig.7 Magnitude and Phase characteristic of $H_{1y}(s)$ and $H_{2y}(s)$

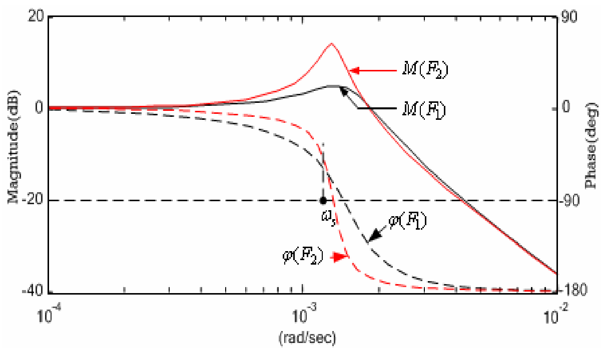

Fig.7 Magnitude and Phase characteristic of $F_{1y}(s)$ and $F_{2y}(s)$

3. Design of azimuth damping network

3.1 Analysis of system error

Although HDN play a good role in damping Schuler oscillation and Foucault oscillation, earth oscillation exists. Usually, accuracy of azimuth gyro drift (ε_z) is lower than horizontal gyro drift (ε_x , ε_y). After INS enter navigation state, change of azimuth gyro drift can cause big azimuth error, then latitude error and longitude error will increase after azimuth error increases[4]. These errors can only modulate by earth oscillation. Equation (11) give system error caused by ε_z :

$$\begin{cases} \gamma = \varepsilon_z \sin\Omega t / \Omega \\ \delta\varphi = -\varepsilon_z (1-\cos\Omega t)\cos\varphi / \Omega \\ \delta\lambda = \varepsilon_z (\sin\varphi \sin\Omega t - \Omega \sin\varphi \cdot t)/\Omega \end{cases} \tag{11}$$

From equation (11), it can be seen that azimuth error is mainly dominated by ε_z . The result of azimuth is to cause latitude and longitude error. In order to solve this problem, external velocity information and azimuth damping network (ADN) will be used to eliminate azimuth, latitude and longitude error.

3.2 Design of azimuth damping network

Fig.9 gives Error block diagram of inertial gyrocompass system which involves HDN and ADN.. By designing optimal ADN, earth oscillation can be destroyed.

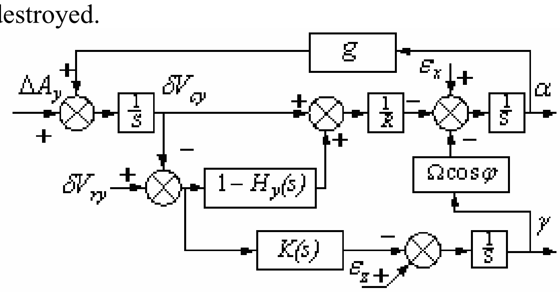

Fig.9 Error block diagram of inertial gyrocompass system

The same as HDN, ADN would have a good characteristic if we take the following into account during the course of choice.

1) HDN in inertial gyrocompass system should following the former four principles, because inertial gyrocompass should realize navigation function.
2) The second principle is that AND ought to damp earth oscillation.
3) The third principle is that ADN must make the system stabilized.
4) In order to make the navigation system is less sensitive to vehicle movement, damping coefficient of whole system is also less than 0.5.

According to Fig.9, expression of $\gamma(s)$ is

$$\gamma(s) = \frac{Rs^2 + gH_y(s)}{Rs^3 + gH_y(s)s + gR\Omega\cos\varphi K(s)}\varepsilon_z(s) \tag{12}$$

Here

$$H_y(s) = (s+\omega_5)(s+\omega_8)/[(s+\omega_6)(s+\omega_7)]$$

$$K(s) = \frac{K_1}{R\Omega\cos\varphi K_2} = \frac{K}{R\Omega\cos\varphi}$$

$$M_1(s) = s^4 + (\omega_6+\omega_7)s^3 + (\omega_6\omega_7+\omega_s^2)s^2 + \omega_s^2(\omega_6+\omega_7)s + \omega_s^2\omega_6\omega_7$$

$$M_2(s) = s^2 + (\omega_6+\omega_7)s + \omega_6\omega_7$$

$$M_3(s) = s^4 + (\omega_6+\omega_7)s^3 + (\omega_6\omega_7+\omega_s^2)s^2 + \omega_s^2(\omega_5+\omega_8)s + \omega_s^2\omega_5\omega_8$$

Characteristic expression of equation (12) is :

$$\Delta(s) = s^5 + (\omega_6+\omega_7)s^4 + (\omega_6\omega_7+\omega_s^2)s^3 + \omega_s^2(\omega_5+\omega_8+K)s^2 + [\omega_s^2\omega_5\omega_8 + \omega_s^2K(\omega_6+\omega_7)]s + \omega_s^2K\omega_6\omega_7 .$$

According to third principle, azimuth damping network must make the system stable. That is to say, all roots of $\Delta(s)$ are in left middle plane. Using Lawes Stabilizing Criterion can solve this problem.

The Lawes Stabilizing expression is

s^5	1	$\omega_6\omega_7+\omega_s^2$	$\omega_s^2(\omega_5\omega_8+K(\omega_6+\omega_7))$
s^4	$\omega_6+\omega_7$	$\omega_s^2(\omega_5+\omega_8+K)$	$\omega_s^2\omega_6\omega_7K$
s^3	b_0	b_1	b_2
s^2	c_0	c_1	c_2
s^1	d_0	d_1	d_2
s^0	e_0	e_1	e_2

By using ω_5 , ω_6 ω_7 , ω_8 ($\xi = 0.5$) , parameter calculation of Lawes table can be get:

$$\begin{cases} b_0 = 3.5432\times10^{-5} - 1.0273\times10^{-5}K \\ b_1 = 5.9011\times10^{-11} + 1.8990\times10^{-8}K \\ b_2 = 0 \end{cases}$$

$$\begin{cases} c_0 = \dfrac{1.5271\times10^{-12} - 2.3636\times10^{-10}K - 1.5771\times10^{-10}K^2}{3.5432\times10^{-5} - 1.0273\times10^{-4}K} \\ c_1 = 5.9022\times10^{-11}K \\ c_2 = 0 \end{cases}$$

$$\begin{cases} d_0 = \dfrac{9.0115\times10^{-23} - 5.9048\times10^{-20}K - 4.0681\times10^{-18}K^2 - 3.6179\times10^{-18}K^3}{1.5271\times10^{-12} - 2.3636\times10^{-10}K - 1.5771\times10^{-10}K^2} \\ e_0 = 1.2280\times10^{-10}K \\ d_1 = d_2 = e_1 = e_2 = 0 \end{cases} \quad (13)$$

In Lawes table, if we can make sure $b_0 > 0$, $c_0 > 0$, $d_0 > 0$, $e_0 > 0$, then the inertial gyrocompass system is absolutely stable. Calculation result of equation (13) is

$$\begin{cases} b_0 > 0 \Rightarrow & K < 0.3449 \\ c_0 > 0 \Rightarrow & -1.50509 < K < 0.006433 \\ d_0 > 0 \Rightarrow & -0.01612 < K < 0.001392 or K < \text{-}1.10969 \\ e_0 > 0 \Rightarrow & K > 0 \end{cases}$$

Then the range of K is $0 < K < 0.001392$.

The optimal value of K should be find from it choice range. The main criterion is oscillation period and damping coefficient of the whole system. Influenced by HDN, the whole system can not reach the same characteristic as the standard system. We can only try our best to make the system to approach the standard system by modulating the value of K.

In order to improve the filter characteristic of the system, $K(s)$ is usually designed 1-order inertial network as $K(s) = K_1 / [R\Omega\cos\varphi(s + K_2)]$.

By continuous modulation of K_1 and K_2, an optimal result is given as equation (14):

$$K_1 = 9.02\times10^{-6}, K_2 = 1.64\times10^{-2} \quad (14)$$

Fig.10 gives the frequency and time domain output of inertial gyrocompass system.

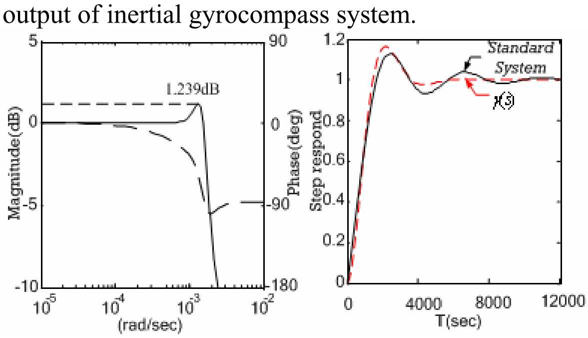

Fig.10 Frequency and time domain output of AND

According to equation (12), the stable output of $\gamma(s)$ is:

$$\gamma = -\frac{\delta V_{ry}}{R\Omega\cos\varphi} + \frac{\varepsilon_x}{\Omega\cos\varphi} \quad (15)$$

From equation (15), it can be seen that accuracy of azimuth error is dominated by ε_x and δV_{ry}. Thus, high accuracy system can be realized despite of low accuracy azimuth gyro.

4. COMPUTER SIMULATION

4.1 Simulation Condition

Initial angle error: $\alpha_0 = \beta_0$=30", γ_0 =2';

Initial position: φ_0=42.3°, λ_0 = 121°;

External velocity error: $\delta V_{rx} = \delta V_{ry} = 0.0\ kn$;

Gyro drift: $\varepsilon_x = \varepsilon_y$=0.002°/h, ε_z=0.004°/h;

4.2 Simulation Figure

According with the given condition, error curve of azimuth angle error and northern velocity error are shown in Fig.11

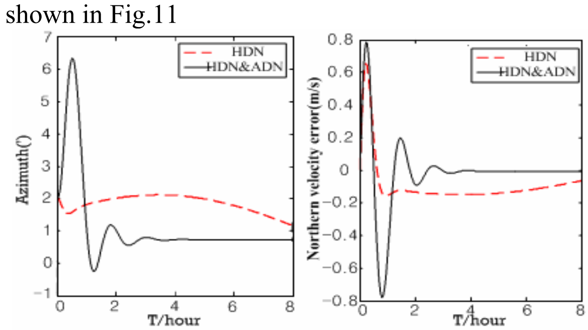

Fig.11 Error curve of azimuth angle error and northern velocity error

4.3 Simulation Conclusion

1) From computer simulation result, it can be find that Schuler oscillation, Schuler oscillation and Foucault oscillation disappeared. That is to say, HDN have played a good role in damping the horizontal loop of the navigation system.

2) The system has decreased to a large extent after azimuth damping network is imported. This means that azimuth damping network has destroyed earth oscillation and the result is the increase of system accuracy.

5. EXPRIMENTATIONS

5.1 Experimentation condition

System: Gimballed Marine Inertial Navigation System composed by Gimballed platform, float gyro, float accelerometer.

5.2 Experimentation Result

1) Swingy experimentation Result

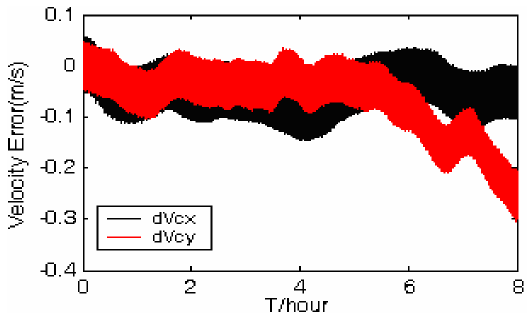

Fig.12 velocity and position error of indoors vibrational experimentation

2) Mooring experimentation Result

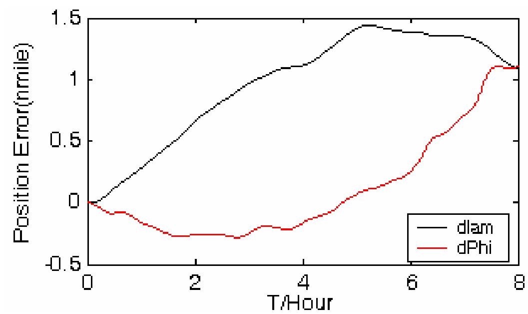

Fig.13 velocity and position error of indoors mooringl experimentation

3) Voyage experimentation Result

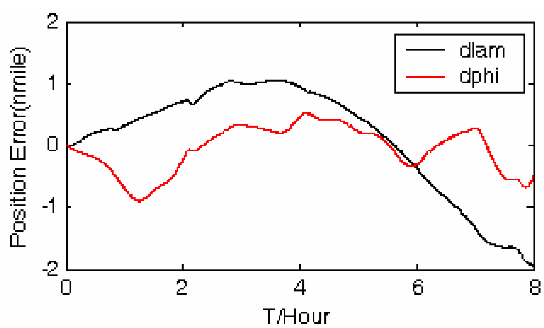

Fig.14 voyage track and position error of voyage experimentation

Here, dVcx is eastern velocity error, dVcy is northern velocity error, dphi is error of latitude, dlam is error of longitude. In addition, external velocity and position information are provided by GPS receiver which style is MS860 of Trimble Corporation.

6. CONCLUSION

In this paper, inertial gyrocompass system is designed to improve positioning accuracy of INS. This make impossible to realize high accuracy INS by using low accuracy gyro and moderate log. Theoretic analyse, computer simulation and experimentation result show that inertial gyrocompass system scheme is effective and practicable.

7. ACKNOWLEDGMENT

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