Chapter 10

Inertial navigation system alignment

10.1 Introduction

Alignment is the process whereby the orientation of the axes of an inertial navigation system is determined with respect to the reference axis system. The basic concept of aligning an inertial navigation system is quite simple and straight forward. However, there are many complications that make alignment both time consuming and complex. Accurate alignment is crucial, however, if precision navigation is to be achieved over long periods of time without any form of aiding.

In addition to the determination of initial attitude, it is necessary to initialise the velocity and position defined by the navigation system as part of the alignment process. However, since it is the angular alignment which frequently poses the major difficulty, this chapter is devoted largely to this aspect of the alignment process.

In many applications, it is essential to achieve an accurate alignment of an inertial navigation system within a very short period of time. This is particularly true in many military applications, in which a very rapid response time is often a prime requirement in order to achieve a very short, if not zero, reaction time.

There are two fundamental types of alignment process: self-alignment, using gyrocompassing techniques, and the alignment of a slave system with respect to a master reference. There are various systematic and random errors that limit the accuracy to which an inertial navigation system can be aligned, whichever method is used. These include the effects of inertial sensor errors, data latency caused by transmission delays, signal quantisation, vibration effects and other undesirable or unquantifiable motion.

Various techniques have been developed to overcome the effects of the random and systematic errors and enable slave systems in missiles, for example, to be aligned whilst under the wing of an aircraft in-flight, or in the magazine of a ship under-way on the ocean. Differing techniques, such as angular rate matching or velocity matching, can be used to align the slave system, the actual circumstances determining the technique which produces the more accurate alignment. In general, a manoeuvre of the aircraft or ship speeds up the alignment process and increases the accuracy achieved.

The basic principles of alignment on both fixed and moving platforms are described in Section 10.2, whilst the particular problems encountered when aligning on the ground, in the air and at sea are discussed in Sections 10.3, 10.4 and 10.5, respectively.

10.2 Basic principles

The inertial system to be aligned contains an instrument cluster in which the gyroscopes and accelerometers are arranged to provide three axes of angular rate information and three axes of specific force data in three directions, which are usually mutually perpendicular. In a conventional sensor arrangement, the sensitive axes of the gyroscopes are physically aligned with the accelerometer axes. Essentially, the alignment process involves the determination of the orientation of the orthogonal axis set defined by the accelerometer input axes with respect to the designated reference frame.

Ideally, we would like the navigation system to be capable of aligning itself automatically following switch-on, without recourse to any external measurement information. In the situation where the aligning system is mounted in a rigid stationary vehicle, a self-alignment may indeed be carried out based solely on the measurements of specific force and angular rate provided by the inertial system as described in the following section.

10.2.1 Alignment on a fixed platform

Consider the situation where it is required to align an inertial navigation system to the local geographic co-ordinate frame defined by the directions of true north and the local vertical. For the purposes of this analysis, it is assumed that the navigation system is stationary with respect to the Earth. In this situation, the accelerometers measure three orthogonal components of the specific force needed to overcome gravity whilst the gyroscopes measure the components of the Earth's turn rate in the same directions.

It is instructive to consider first the alignment of a stabilised platform system in which the instrument cluster can be rotated physically into alignment with the local geographic reference frame. In this situation, it is usual to refer to the accelerometers whose sensitive axes are to be aligned with the north, east and vertical axes of the reference frame as the north, east and vertical accelerometers respectively. Similarly, north, east and vertical gyroscopes may be defined.

In a platform mechanisation, alignment is achieved by adjusting the orientation of the platform until the measured components of specific force and Earth's rate become equal to the expected values. The horizontal components of gravity acting in the north and east directions are nominally zero. The instrument cluster is therefore rotated until the outputs of the north and east accelerometers reach a null, thus levelling the platform. Since the east component of Earth's rate is also known to be zero, the platform is then rotated about the vertical until the east gyroscope output is nulled, thus achieving an alignment in azimuth. This type of process is referred to as gyrocompassing and is described extensively in the literature [1]. An equivalent alignment process, sometimes referred to as analytic gyrocompassing, can be used to align a strapdown inertial navigation system as described next.

In a strapdown system, attitude information may be stored either as a direction cosine matrix or as a set of quaternion parameters, as described in Chapter 3. The objective of the angular alignment process is to determine the direction cosine matrix or the quaternion parameters which define the relationship between the inertial sensor axes and the local geographic frame. The measurements provided by the inertial sensors in body axes may be resolved into the local geographic frame using the current best estimate of the body attitude with respect to this frame. The resolved sensor measurements are then compared with the expected turn rates and accelerations to enable the direction cosines or quaternion parameters to be calculated correctly. The principles of the method are illustrated below with the aid of single plane examples to show how the attitude of the strapdown inertial sensors with respect to the local geographic reference frame may be extracted from the inertial measurements.

Since the true components of gravity in the north and east directions are nominally zero, any departure from zero in the accelerometer measurements resolved in these directions may be interpreted as an error in the stored attitude data, and in particular as an error in the knowledge of the direction of the local vertical. A single plane illustration is given in Figure 10.1.

The accelerometers provide measurements of the true acceleration in body axes, $-g\sin\theta$ and $-g\cos\theta$ respectively. These measurements are resolved through an angle θ' which is an estimate of the true body angle θ, or the angle that the body makes with the estimated reference frame shown in the figure. It can be seen from the figure that the resolved component in the estimated horizontal plane, denoted g_x, is given by:

$$g_x = -g\sin(\theta - \theta') \tag{10.1}$$

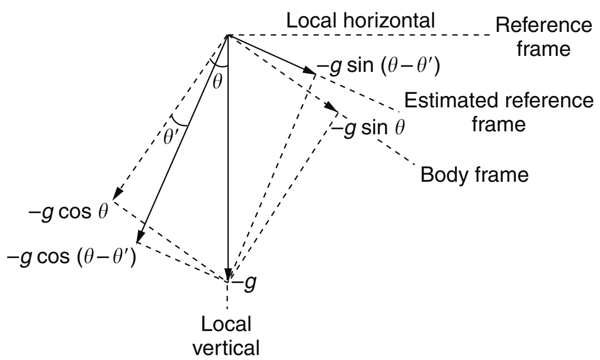

Figure 10.1 Alignment to the gravity vector in a single plane

θ' may be adjusted until g_x becomes zero, at which time $\theta' = \theta$, that is, the estimated body angle becomes equal to the true body angle and the estimated reference frame becomes coincident with the true reference frame.

Given accurate measurements of the specific force acceleration, this process allows the orientation of the axis set defined by the accelerometers with respect to the local vertical to be defined accurately, and is analogous to the process of levelling the stable element in a platform inertial navigation system.

Having defined the local horizontal plane, and so effectively achieved a 'level' in the alignment process, it is then necessary to determine the heading or azimuthal orientation of the inertial instrument frame in the horizontal plane, that is, to determine direction with respect to true north. This is achieved from knowledge of the true components of Earth's rate in the local geographic frame. Assuming that the gyroscopes are of sufficient precision to detect Earth's rate accurately, the stored attitude information is now adjusted until the resolved component of the measured rate in the east direction reduces to zero. A diagram illustrating the alignment in azimuth is shown in Figure 10.2.

In this case, ψ is the true orientation of the x-axis of the instrument frame with respect to true north whilst ψ' is the estimate of that quantity. The components of Earth's rate (Ω) detected by the x- and y-axis gyroscopes shown in the figure are $\Omega \cos L \cos \psi$ and $\Omega \cos L \sin \psi$, respectively, where L is the latitude of the aligning system. The east component of Earth's rate as determined by the navigation system, denoted ω_E, may be expressed as follows:

$$\omega_E = \Omega \cos L \sin(\psi - \psi') \tag{10.2}$$

ψ' is adjusted until ω_E becomes zero, in which case $\psi' = \psi$.

10.2.2 Alignment on a moving platform

In order to align a strapdown inertial navigation system in a moving vehicle, a technique which is similar in principle to that described above may be used.

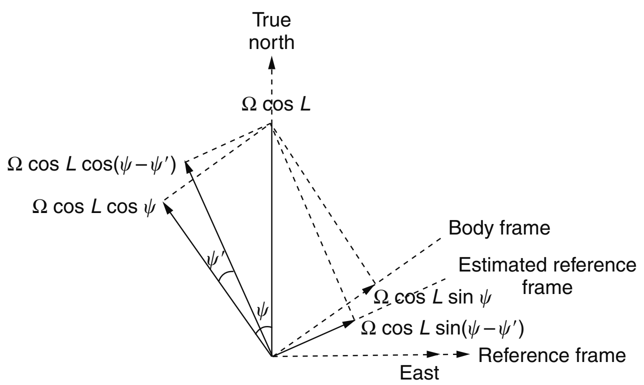

Figure 10.2 Alignment in azimuth

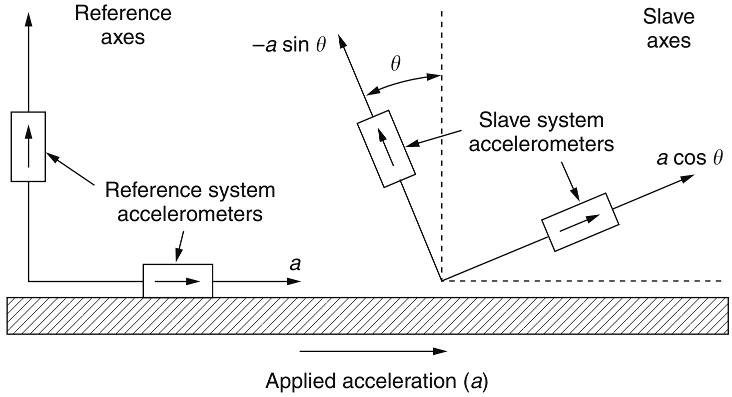

Figure 10.3 Measurement matching alignment in a single plane

However, when aligning in a moving vehicle, the accelerations and turn rates to which the system is subjected are no longer well defined in the way that they are when the system is stationary. It therefore becomes necessary to provide some independent measure of these quantities against which the measurements generated by the aligning system may be compared.

Consider the situation depicted in Figure 10.3 in which the axes defined by the strapdown sensors are shown rotated through an angle θ in a single plane with respect to the navigation reference frame.

If the acceleration of the vehicle in the reference x-direction is a, then the accelerations sensed by the strapdown system accelerometers will be as follows:

$$\begin{aligned} a_x &= a\cos\theta \\ a_y &= -a\sin\theta \end{aligned} \tag{10.3}$$

In the absence of any instrument measurement inaccuracies, alignment of the strapdown system may be achieved by resolving the accelerometer measurements through an angle θ' and adjusting its magnitude using a feedback process so as to null the difference between the resolved components of the slave system measurements and the accelerations measured by the reference system.

Mathematically, θ' is adjusted to allow the following relationships to be satisfied:

$$\begin{aligned} a_x\cos\theta' - a_y\sin\theta' &= a \\ a_x\sin\theta' + a_y\cos\theta' &= 0 \end{aligned} \tag{10.4}$$

Substituting for a_x and a_y from eqn. (10.3) yields:

$$\begin{aligned} a\cos(\theta - \theta') &= a \\ a\sin(\theta - \theta') &= 0 \end{aligned} \tag{10.5}$$

It can be seen that these relationships will be satisfied when $\theta' = \theta$.

Therefore, it is possible to determine the orientation of the strapdown sensors by comparing the accelerometer measurements resolved into the reference frame with independent measurements of these same quantities. An estimate of θ can also be derived in a similar manner by comparing angular rate measurements. Whichever method is adopted, it will be noted that alignment about a given axis is dependent on the measurement of an acceleration or turn rate taking place along or about an axis which is orthogonal to the axis in which the misalignment exists.

As an alternative to the type of procedure described above, alignment may be achieved by comparing estimates of velocity or position generated by the strapdown system with similar estimates provided by an external source over a period of time. Velocity and position errors will propagate with time as a result of the angular alignment errors. Therefore, any difference in the velocity and position estimates generated between the aligning system and the external source over this time will be partially the result of an alignment error. Such methods are discussed in more detail below in the context of in-flight and shipboard alignment.

With aircraft and shipboard systems, the independent measurement information may be provided by a separate inertial navigation system on-board the same vehicle. By comparing the two sets of inertial measurements it is possible to deduce the relative orientation of the two frames on a 'continuous' basis. The precise measurements available will be dependent on the reference system mechanisation on-board the ship or aircraft. As a rule, a stable platform navigation system will only output estimates of position, velocity, attitude and heading. A strapdown reference system offers greater flexibility, potentially providing linear acceleration and angular rate information in addition to the usual navigation outputs listed above. Alternatively, position fixes may be derived on-board the vehicle from signals transmitted by a radio beacon or from satellites.

10.3 Alignment on the ground

10.3.1 Introduction

Attention is now turned to the alignment of an inertial navigation system in a ground based vehicle. Clearly, the scope for carrying out manoeuvres or applying motion to aid the process of alignment is very limited in such applications. Attention is focused here on a requirement which often arises in practice, that of determining the orientation of a set of sensor axes with respect to the local geographic frame. For convenience, the local geographic axis set is often chosen to be the reference frame.

In the past, a site survey would be carried out to establish a north line. Heading information would then be transferred to the aligning navigation system using theodolites and a prism attached to the aligning system. Although high accuracy can be obtained using this approach, it is both time consuming and labour intensive. The methods discussed in the following sections are usually more convenient to implement and avoid such problems.

10.3.2 Ground alignment methods

In principle, the techniques outlined in Section 10.2 for the self-alignment of a strapdown inertial system on a stationary platform can be used. We now look in more detail at the computation required to implement that alignment process. As described above, the objective of the angular alignment process is to determine the direction cosine matrix, $\mathbf{C}_{\mathrm{n}}^{\mathrm{b}}$, or its quaternion equivalent, which relates the body and geographic reference frames. The body mounted sensors will measure components of the specific force needed to overcome gravity and components of Earth's rate, denoted by the vector quantities $\mathbf{g}^{\mathrm{b}}$ and $\boldsymbol{\omega}_{\mathrm{ie}}^{\mathrm{b}}$, respectively. These vectors are related to the gravity and Earth's rate vectors specified in the local geographic frame, $\mathbf{g}^{\mathrm{n}}$ and $\boldsymbol{\omega}_{\mathrm{ie}}^{\mathrm{n}}$, respectively, in accordance with the following equations:

$$\mathbf{g}^{\mathrm{b}} = \mathbf{C}_{\mathrm{n}}^{\mathrm{b}}\mathbf{g}^{\mathrm{n}} \tag{10.6}$$

$$\boldsymbol{\omega}_{\mathrm{ie}}^{\mathrm{b}} = \mathbf{C}_{\mathrm{n}}^{\mathrm{b}}\boldsymbol{\omega}_{\mathrm{ie}}^{\mathrm{n}} \tag{10.7}$$

where $\mathbf{g}^{\mathrm{n}} = [0 \quad 0 \quad -g]^{\mathrm{T}}$ and $\boldsymbol{\omega}_{\mathrm{ie}}^{\mathrm{n}} = [\Omega\cos L \quad 0 \quad -\Omega\sin L]^{\mathrm{T}}$ in which Ω and L denote Earth's rate and latitude, respectively. Given knowledge of these quantities, estimates of the elements of the direction cosine matrix may be computed directly from the measurements of $\mathbf{g}^{\mathrm{b}} = [g_x \quad g_y \quad g_z]^{\mathrm{T}}$ and $\boldsymbol{\omega}_{\mathrm{ie}}^{\mathrm{b}} = [\omega_x \quad \omega_y \quad \omega_z]^{\mathrm{T}}$ as follows:

$$\begin{aligned} c_{31} &= -\frac{g_x}{g} \qquad c_{11} = \frac{\omega_x}{\Omega\cos L} - \frac{g_x\tan L}{g} \\ c_{32} &= -\frac{g_y}{g} \qquad c_{12} = \frac{\omega_y}{\Omega\cos L} - \frac{g_y\tan L}{g} \\ c_{33} &= -\frac{g_z}{g} \qquad c_{13} = \frac{\omega_z}{\Omega\cos L} - \frac{g_z\tan L}{g} \end{aligned} \tag{10.8}$$

where $c_{11}, c_{12}, \ldots, c_{33}$ are elements of the direction cosine matrix $\mathbf{C}_{\mathrm{n}}^{\mathrm{b}}$. The remaining direction cosine elements (c_{21}, c_{22} and c_{23}) may be determined by making use of the orthogonality properties of the direction cosine matrix which yield:

$$\begin{aligned} c_{21} &= -c_{12}c_{33} + c_{13}c_{32} \\ c_{22} &= c_{11}c_{33} - c_{31}c_{13} \\ c_{23} &= -c_{11}c_{32} + c_{31}c_{12} \end{aligned} \tag{10.9}$$

It can be seen from the above equations that the direction cosine matrix is uniquely defined provided that L is not equal to $\pm 90°$, that is, there is a unique value so long as the aligning system is not located at either the north or south poles of the Earth. This clearly would lead to singularities in the equations for some of the direction cosine elements which therefore become indeterminate. However, over much of the Earth's surface, a single set of inertial measurements can provide all of the information needed to compute the direction cosine matrix, and so achieve a strapdown system alignment.

The accuracy with which such an alignment can be accomplished is largely determined by the precision of the available measurements and the resolution of the instrument outputs. As a result of instrument biases, the above procedure will yield an estimate of the direction cosine matrix $\tilde{\mathbf{C}}_b^n$ which will be in error. As described in Chapter 11, $\tilde{\mathbf{C}}_b^n$ may be expressed as the product of the true matrix $\mathbf{C}_b^n$ and a matrix $\mathbf{B}$ which represents the misalignment between the actual and computed geographic frames:

$$\tilde{\mathbf{C}}_b^n = \mathbf{B}\,\mathbf{C}_b^n \tag{10.10}$$

For small angular misalignments, this can be written in skew symmetric form as:

$$\mathbf{B} = \mathbf{I} - \boldsymbol{\Psi} \tag{10.11}$$

where $\mathbf{I}$ is a 3×3 identity matrix and

$$\boldsymbol{\Psi} = \begin{pmatrix} 1 & -\delta\gamma & \delta\beta \\ \delta\gamma & 1 & -\delta\alpha \\ -\delta\beta & \delta\alpha & 1 \end{pmatrix} \tag{10.12}$$

$\delta\alpha$, $\delta\beta$ and $\delta\gamma$ are the misalignments about the north, east and vertical axes of the geographic frame, respectively, and are equivalent to the physical misalignments of the instrument cluster in a stable platform navigation system. The 'tilt' errors ($\delta\alpha$ and $\delta\beta$) which result, are predominantly determined by the accelerometer biases while the azimuth or heading error ($\delta\gamma$) is a function of gyroscopic bias as described in the following section.

The direction cosine matrix, $\tilde{\mathbf{C}}_b^n$, is adjusted through the alignment process until the residual north and east components of accelerometer bias are off-set by components of g in each of these directions, effectively nulling the estimates of acceleration in these directions. The resulting attitude errors correspond to the 'tilt' errors which arise when aligning a stable platform system. In azimuth, the platform rotates about the vertical to a position where a component of the Earth's horizontal rate ($\Omega \cos L$) appears about the east axis to null the east gyroscopic bias. An equivalent process takes place in a strapdown system, again through appropriate adjustment of the direction cosine matrix.

The resulting attitude and heading errors may be expressed as follows for the particular situation in which the body frame is nominally aligned with the geographic frame, that is, where $\mathbf{C}_b^n = \mathbf{I}$, it can be shown that:

$$\begin{aligned} \delta\alpha &= \frac{B_y}{g} \\ \delta\beta &= -\frac{B_x}{g} \\ \delta\gamma &= \frac{D_y}{\Omega \cos L} + \frac{B_y \tan L}{g} \end{aligned} \tag{10.13}$$

More generally, where the system is not aligned with the geographic frame, the sensor biases arising in each of the above equations will be made up of a linear combination of the biases in all three gyroscopes or all three accelerometers.

10.3.2.1 Derivation of azimuth error, $\delta\gamma$

The angular rates sensed about the x-, y- and z-axes may be expressed in vector form, as the sum of the Earth's rate components in each axis and the residual gyroscope biases, as follows:

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 1 & -\delta\gamma & \delta\beta \\ \delta\gamma & 1 & -\delta\alpha \\ -\delta\beta & \delta\alpha & 1 \end{bmatrix} \begin{bmatrix} \Omega \cos L \\ 0 \\ -\Omega \sin L \end{bmatrix} + \begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix}$$

The process of gyrocompassing acts to null the east component of measured angular rate; the ω_y term:

$$\omega_y = \delta\gamma\Omega \cos L + \delta\alpha\Omega \sin L + D_y = 0$$

Substituting for $\delta\alpha$ from eqn. (10.13) and rearranging yields,

$$\delta\gamma = \frac{D_y}{\Omega \cos L} + \frac{B_y \tan L}{g}$$

as given above.

The azimuth misalignment term ($\delta\gamma$) contains two components; the first being the result of a residual gyroscopic bias acting in the east direction, the second term being the result of a level or tilt error about the north axis causing a component of vertical Earth's rate ($\delta\alpha\Omega \sin L$) to appear as a further bias about the east axis.

It can be shown using eqn. (10.13) that a 1 milli-g accelerometer bias will give rise to a level error of 1 mrad ($\sim$3.4 arc min) whilst a gyroscopic drift of 0.01°/h will result in an azimuthal alignment error of 1 mrad at a latitude of 45°. The relationship between gyroscope bias and azimuthal error is illustrated graphically in Figure 10.4. It is clear that good quality gyroscopes are needed to achieve an accurate alignment in azimuth. It is noted that for some inertial system applications, it is the alignment requirements which can dictate the specification of the inertial sensors rather than the way in which the sensor errors propagate during navigation.

The alignment method as described here, using a single set of instrument measurements, would allow only a coarse alignment to take place. To achieve a more accurate estimate of the direction cosine matrix, sequential measurements would be used to carry out a self-alignment over a period of time. Some Kalman filtering of the measurement data would normally be applied under these circumstances.

In addition to the alignment error mechanisms described above, errors in azimuth also arise as a result of gyroscopic random noise (n) and accelerometer bias instability (b). Noise on the output of the gyroscopes (random walk in angle), which is of particular concern in systems using mechanically dithered ring laser gyroscopes, gives rise to a root mean square azimuth alignment error which is inversely proportional to the square root of the alignment time (t_a), viz. $\delta\gamma = n/\Omega \cos L\sqrt{t_a}$. Therefore, given a random walk error of $0.005°/\sqrt{\text{h}}$, an alignment accuracy of 1 mrad can be achieved at a latitude of 45° in a period of 15 min. The effect of this noise can be reduced by extending the alignment period, that is, extending the time over which the noise is filtered. Small changes in the north component of accelerometer bias (b) with time are equivalent to an east gyroscope drift. Therefore such errors can also

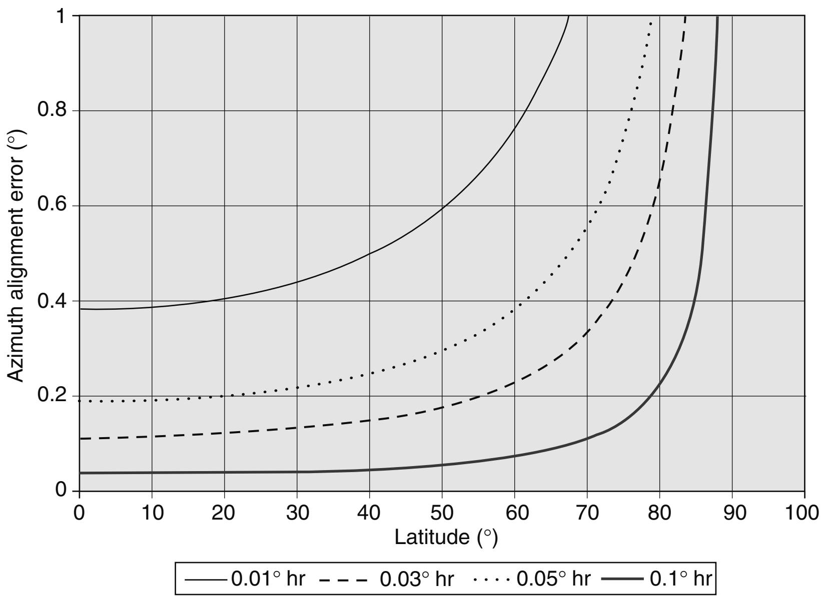

Figure 10.4 Azimuth alignment error versus latitude as a function of residual east gyroscope bias

introduce an azimuth alignment error which may be expressed as $\delta\gamma = b/g\Omega \cos L$. A bias drift of 1 micro-g/s will result in an alignment error of 20 mrad at a latitude of 45°. The minimisation of bias shifts with temperature as well as switch-on transients is vital for applications where this effect becomes significant.

10.3.2.2 Vehicle perturbations

A process very similar to that described above may be adopted to align an inertial navigation system mounted in a vehicle which is not perfectly stationary, but subjected to disturbances. For instance, it may be required to align a navigation system in an aircraft on a runway preparing for take-off which is being buffeted by the wind and perturbed by engine vibration. In such a situation, the mean attitude of the aligning system with respect to the local geographic frame is fixed, and the specific force and turn rates to which the aligning system is subjected are nominally fixed. In this situation, some form of base motion isolation is needed to allow the alignment errors to be deduced from the measurements of turn rate and specific force provided by the sensors [1].

A self-alignment may be carried out in the presence of the small perturbations using a Kalman filter incorporating a model of the base motion disturbance. Failure to take account of any filter measurement differences caused by the disturbances will result in an incorrect alignment, since the measurements of the disturbance will be interpreted incorrectly as resulting from alignment errors. The application of Kalman filtering techniques for the alignment of strapdown inertial navigation systems is discussed more fully in Sections 10.4 and 10.5 in relation to the alignment of such systems in-flight and at sea.

10.3.3 Northfinding techniques

In view of the limitations of both of the aforementioned techniques, various designs for special purpose equipment, which would allow the directions of the local vertical and true north to be defined within a land-based vehicle, have been produced. Such devices, often referred to as northfinders, are designed with a view to establishing the direction of true north within a short period of time using relatively inexpensive inertial sensors.

One possible mechanisation uses measurements of two orthogonal components of Earth's rate to establish a bearing angle of a pre-defined case reference axis with respect to north. The sensing element is a two-degrees-of-freedom gyroscope such as a dynamically tuned gyroscope (DTG) with its spin axis vertical. The DTG assembly is suspended by a wire to provide automatic levelling of the two input axes which are at right angles to one another. Hence, the input axes are maintained in the horizontal plane. The input axes are held in a torque re-balance loop to provide measurements of the rate of turn about each axis. The pendulous assembly is enclosed within a container which is filled with a fluid to provide damping.

In this configuration, the gyroscope measures two horizontal components of the Earth's rotation rate as indicated in Figure 10.5.

The angular rates (ω_x and ω_y) measured about the two input axes of the gyroscope may be expressed as follows:

$$\begin{aligned} \omega_x &= \Omega \cos L \cos \psi \\ \omega_y &= \Omega \cos L \sin \psi \end{aligned} \qquad (10.14)$$

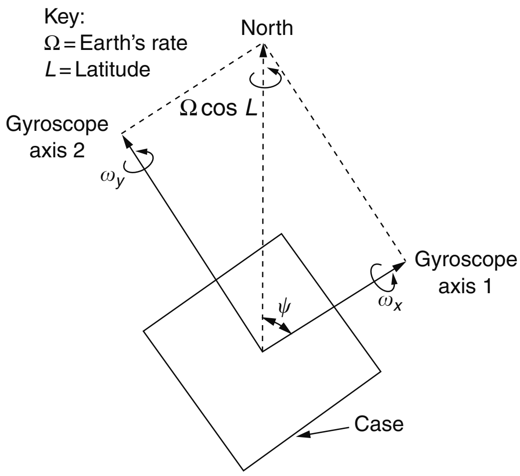

Figure 10.5 A northfinder

where Ω is the Earth's rate, L is the latitude and ψ is the heading of gyroscope axis with respect to true north.

By taking the ratio of the two independent gyroscopic measurements, the latitude dependent terms cancel, allowing the gyroscope heading angle, ψ, to be computed.

$$\frac{\omega_y}{\omega_x} = \frac{\Omega \cos L \sin \psi}{\Omega \cos L \cos \psi} = \tan \psi$$

$$\psi = \arctan\left(\frac{\omega_y}{\omega_x}\right) \tag{10.15}$$

Heading can be calculated in this way provided $\omega_x \neq 0$. In the event that ω_x is close to zero, the following equation may be used:

$$\psi = 90 - \arctan\left(\frac{\omega_x}{\omega_y}\right) \tag{10.16}$$

It can be seen that the northfinder does not require knowledge of latitude, or prior orientation in any particular direction, to enable a measure of heading to be obtained.

In order to achieve useful accuracy from a device of this type, gyroscope measurement accuracy of 0.005°/h or better may be required. However, the need for a highly accurate gyroscope may be avoided by rotating the entire sensor assembly through 180° about the vertical, without switching off, and then taking a second pair of measurements in this new orientation. The measurements obtained in each position are then differenced, allowing any biases on the measurements to be largely eliminated. The heading angle is then computed from the ratio of the measurement differences. This process is identical to the 'indexing technique' used in inertial systems to enhance accuracy.

The rotation of the sensor may be accomplished using a small d.c. motor to drive the assembly from one mechanical stop to another which are nominally 180° apart. The stops are positioned so that the gyroscope input axes are aligned with the case reference axis, or at right angles to it, when the measurements are taken. Over the short period of time required to rotate the sensor (typically 5 s) and to take these measurements, all but the gyroscope in-run random measurement errors can be removed. This technique also helps to reduce any errors arising through the sensitive axes of the gyroscope not being perfectly horizontal.

There are a number of variations of this method, one of which involves positioning the gyroscope with one of its input axes vertical and the spin axis in the horizontal plane. Two measurements of the horizontal component of Earth's rate are taken with the gyroscope in two separate orientations 90° apart. An estimate of heading can then be obtained from the ratio of these two measurements in the manner described above. This scheme allows the Northfinder to be used as a directional gyroscope after the heading angle has been determined. Other variations incorporate accelerometers to allow the inclination to the vertical to be determined as well as heading.

10.4 In-flight alignment

10.4.1 Introduction

The requirement frequently arises to align an inertial navigation system in an air-launched missile prior to missile release from an aircraft platform. A convenient reference for this purpose may be provided by the aircraft's own inertial navigation system. Such an alignment of the missile system may therefore be achieved by the transfer of data from the aircraft's navigation system to the missile by a process known as transfer alignment. This may be achieved quite simply by the direct copying of data from the aircraft to the missile navigation system, or more precisely by using some form of inertial measurement matching process of the type outlined in Section 10.2.2. Alternatively, the missile inertial navigation system may be aligned in-flight using position fixes provided by satellite or airborne radar systems. All such methods are discussed below, but with particular emphasis on the use of transfer alignment.

It is noted that it is sometimes neither desirable nor possible to have the inertial system in a guided missile 'run-up' and aligned waiting for the launch command. In this situation, it is required to align the missile's inertial navigation system very rapidly, immediately prior to launch of the missile.

10.4.2 Sources of error

As a result of physical misalignments between different mounting locations on an aircraft, the accuracy with which inertial data can be transferred from one location to another on-board the aircraft will be restricted. Such errors may be categorised in terms of static and dynamic components as follows:

Static errors will exist as a result of manufacturing tolerances and imprecise installation of equipment leading to mounting misalignments between different items of equipment on the aircraft.

Dynamic errors will exist because the airframe will not be perfectly rigid and will bend in response to the aerodynamic loading on the wings and launch rails to which a missile is attached. Such effects become particularly significant in the presence of aircraft manoeuvres. Significant error contributions can also be expected to arise as a result of vibration.

Methods of alleviating such problems are discussed in the following section.

10.4.3 In-flight alignment methods

Attention is focused here on the alignment of an inertial navigation system contained in an air-launched missile which may be attached to a fuselage or wing pylon beneath a 'carrier' aircraft.

10.4.3.1 'One-shot' transfer alignment

One of the simplest alignment techniques which may be adopted in this situation is to copy position, velocity and attitude data from the aircraft's own navigation system

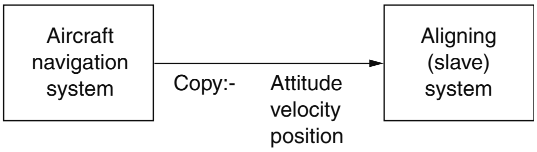

Figure 10.6 'One-shot' transfer alignment

directly to the missile system. This is sometimes referred to as a 'one-shot' alignment process and is depicted in Figure 10.6.

Clearly, any angular displacement between the aircraft and missile systems which exists at the instant when the data are transferred will appear as an alignment error in the missile's navigation system. Therefore, the success of such a scheme is reliant on the two systems being physically harmonised to high accuracy, or on accurate knowledge of their relative orientation being available when the alignment takes place. In the latter situation, the data from the aircraft's navigation system may be resolved accurately in missile axes before being passed to the missile navigation system.

In general, the precise harmonisation of one system with respect to the other will not be known, for the reasons outlined in the previous section. Furthermore, the aircraft navigation system will be positioned some distance from the aligning system in the missile and there will be relative motion between them should the aircraft turn or manoeuvre; the so-called lever-arm motion. In this situation, the velocity information passed to the missile will be in error. As a result, the accuracy of alignment which can be achieved using a 'one-shot' alignment procedure will be extremely limited and more precise methods are usually sought.

10.4.3.2 Airborne inertial measurement matching

An alternative method of transfer alignment, which has received much attention in recent years [2–5], is that of inertial measurement matching. This technique relies on the comparison of measurements of applied motion obtained from the two systems to compute the relative orientation of their reference axes, as introduced in the discussion of basic principles in Section 10.2 and depicted in Figure 10.7. An initial coarse alignment may be achieved by the 'one-shot' process, discussed earlier, before initiating the measurement matching process which is described below.

In theory, a transfer alignment between two inertial navigation systems on an aircraft can be achieved most rapidly by comparing measurements generated by the aircraft system and the missile system of the fundamental navigation quantities of specific force acceleration and angular rate, resolved into a common co-ordinate frame. In the absence of measurement errors, and assuming the two systems are mounted side by side on a perfectly rigid platform, the measurement differences arise purely as a result of alignment errors. Under such conditions, it is possible to identify accurately the misalignments between the two systems.

In practice, this approach is often impractical for a number of reasons. The reference system may use 'platform' technology, in which linear acceleration and turn

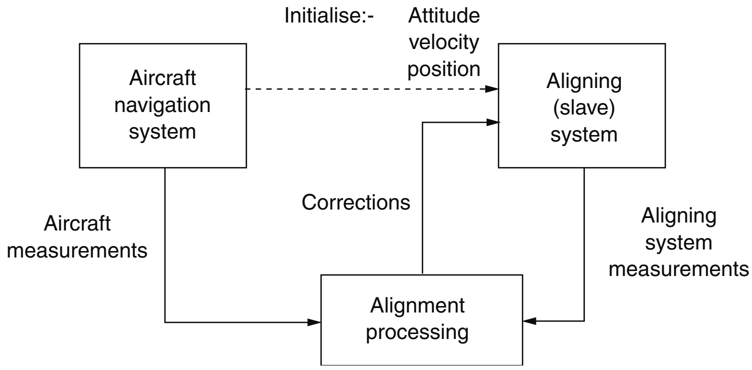

Figure 10.7 Inertial measurement matching alignment scheme

rate data are not standard outputs. This is particularly true in the case of many older military aircraft, although the situation has changed with the wider use of strapdown technology in modern combat aircraft inertial navigation systems.

There are also technical reasons which may preclude the use of linear acceleration and angular rate matching procedures as a viable option for airborne transfer alignment. This is particularly true where the physical separation between the reference and aligning system is large, and where significant flexure motion is present. The turn rates and linear accelerations sensed by the reference and aligning systems will differ as a result of the flexure motion which is present. These differences will then be interpreted incorrectly as errors in the stored attitude data, and so degrade the accuracy of alignment which can be achieved. Acceleration matching and angular rate matching are particularly sensitive to the effects of flexure. Whilst it is possible theoretically to model the flexural motion, and thus separate the components of the measurement differences caused by flexure from those attributable to alignment errors, adequate models of such motion are rarely available in practice.

Even when attempting to carry out an alignment on a perfectly rigid airframe, the translational motion sensed at the reference and the aligning system locations will differ, as the aircraft rotates, as a result of lever-arm motion. The measurement differences which arise as a result of lever-arm motion as the aircraft manoeuvres will also be interpreted incorrectly as alignment inaccuracies and therefore inhibit the alignment process. These additional measurement differences are functions of aircraft turn rate, angular acceleration and the physical separation between the two systems. Whilst it is theoretically possible to correct one set of measurements before comparison with the other, such corrections are dependent on the availability of sufficiently precise estimates of these quantities. Although it is reasonable to assume that distance would be known to sufficient accuracy and the actual turn rates may be provided directly by a strapdown system, angular acceleration measurements are not usually available and without the use of angular accelerometers are not easy to estimate.

For the reasons outlined above, acceleration and rate matching are not generally recommended for alignment of inertial systems on-board aircraft, even when both the reference and aligning systems are configured in a strapdown form. An alternative approach is the use of velocity matching described in Section 10.4.3.3. Velocity errors propagate in an inertial navigation system as a result of alignment inaccuracy, as well as through inertial instrument imperfections. By comparing the velocity estimates provided by the reference and aligning systems, it may therefore be possible to obtain estimates of the alignment errors and, under some circumstances, estimates of the sensor biases. Hence, it is possible to achieve a measure of sensor calibration as part of the same process.

Because of the smoothing effect of the integration process which takes place between the raw measurements from the instruments and the velocity estimates within an inertial navigation system, the effects of flexure and sensor noise on the process of alignment is much less severe than experienced with acceleration matching. Further, it has the advantage of allowing lever-arm corrections to be implemented more easily, such corrections at the 'velocity' level being purely functions of turn rate and separation distance.

10.4.3.3 Velocity matching alignment

As suggested in the preceding section, an in-flight alignment may be achieved by comparing estimates of velocity generated by the aligning system with estimates of the same quantities provided by the aircraft's own navigation system. The nature of the alignment problem, which involves the identification of a number of interrelated and time varying error sources using measurements which are corrupted with noise, is well suited to statistical modelling techniques. These techniques include Kalman filtering, the principles of which are discussed in Appendix A.

This section outlines the system and measurement equations required to construct a Kalman filter which may be used to process the velocity information and so obtain estimates of the alignment errors. For the purposes of this Kalman filter illustration, a number of simplifying assumptions have been made in the formulation given here and these are described below.

The system equations

It is required to determine accurately the attitude and velocity of the aligning system with respect to a designated reference frame. Typically, this may be a body fixed axis set within the aircraft or the local geographic navigation frame. The aligning system and reference frames are denoted here by the superscripts and subscripts b and n, respectively. Following the notation used in Chapter 3, the propagation of the direction cosine matrix ($\mathbf{C}_b^n$) which relates the sensor axes of the aligning system to the reference frame is governed by the following differential equation:

$$\dot{\mathbf{C}}_b^n = \mathbf{C}_b^n \mathbf{\Omega}_{nb}^b \tag{10.17}$$

where $\mathbf{\Omega}_{nb}^b$ is a skew symmetric matrix formed from the turn rates of the aligning system with respect to the reference frame. This turn rate is obtained by differencing the angular rates sensed by the aligning system ($\boldsymbol{\omega}_{ib}^{b}$) and the turn rate of the reference frame ($\boldsymbol{\omega}_{in}^{n}$). An estimate of the direction cosine matrix, denoted $\hat{\mathbf{C}}_{b}^{n}$, is calculated using measurements of the turn rate to which the aligning system is subjected ($\hat{\boldsymbol{\omega}}_{ib}^{b}$) and an estimate of the reference frame rate ($\hat{\boldsymbol{\omega}}_{in}^{n}$) to determine $\hat{\boldsymbol{\Omega}}_{nb}^{b}$, updating from some initial estimate using:

$$\dot{\hat{\mathbf{C}}}_{b}^{n} = \hat{\mathbf{C}}_{b}^{n}\hat{\boldsymbol{\Omega}}_{nb}^{b} \tag{10.18}$$

As described in Section 10.3.2, for small angle misalignments, the true and estimated direction cosine matrices may be related by the equation:

$$\hat{\mathbf{C}}_{b}^{n} = [\mathbf{I} - \boldsymbol{\Psi}]\mathbf{C}_{b}^{n} \tag{10.19}$$

where $\mathbf{I}$ is the identity matrix and $\boldsymbol{\Psi}$ is a skew symmetric matrix which may be written as:

$$\boldsymbol{\Psi} = \begin{pmatrix} 0 & -\delta\gamma & \delta\beta \\ \delta\gamma & 0 & -\delta\alpha \\ -\delta\beta & \delta\alpha & 0 \end{pmatrix}$$

in which the off-diagonal elements $\delta\alpha$, $\delta\beta$ and $\delta\gamma$ represent the attitude errors in the aligning system.

It can be shown that the attitude errors propagate according to:

$$\dot{\boldsymbol{\Psi}} = -\boldsymbol{\omega}_{in}^{n} \times \boldsymbol{\Psi} - \mathbf{C}_{b}^{n}\delta\boldsymbol{\omega}_{ib}^{b} + \delta\boldsymbol{\omega}_{in}^{n} \tag{10.20}$$

where $\boldsymbol{\Psi} = [\delta\alpha \quad \delta\beta \quad \delta\gamma]^{T}$, is the alignment error vector; $\delta\boldsymbol{\omega}_{ib}^{b} = (\tilde{\boldsymbol{\omega}}_{ib}^{b} - \boldsymbol{\omega}_{ib}^{b})$ is the gyroscopic measurement error in the aligning system; $\delta\boldsymbol{\omega}_{in}^{n} = (\tilde{\boldsymbol{\omega}}_{in}^{n} - \boldsymbol{\omega}_{in}^{n})$ is the error in the reference frame rate estimates and $\times$ denotes the cross product of two vector quantities.

For the purposes of this example Kalman filter formulation, the gyroscopic errors are modelled in the filter as additive Gaussian white noise and the reference rate errors are assumed to be zero. The derivation of this equation is given in Chapter 12 where the propagation of errors in strapdown inertial navigation systems is discussed in greater detail.

The velocity equations may be expressed approximately as:

$$\dot{\mathbf{v}}^{n} = \mathbf{C}_{b}^{n}\mathbf{f}^{b} - \mathbf{g} \tag{10.21}$$

where $\mathbf{v}^{n}$ is the velocity of the aircraft, $\mathbf{f}^{b}$ is the specific force sensed by the accelerometers in the aligning system in body axes and $\mathbf{g}$ is the local gravity vector. The propagation of the errors in the estimates of velocity computed by the aligning system ($\delta\mathbf{v}^{n}$) may be expressed as:

$$\delta\dot{\mathbf{v}}^{n} = \mathbf{f}^{n} \times \boldsymbol{\Psi} + \mathbf{C}_{b}^{n}\delta\mathbf{f}^{b} \tag{10.22}$$

where $\mathbf{f}^{n}$ is the specific force measured by the aligning system resolved in reference axes and $\delta\mathbf{f}^{b}$ represents the errors in the accelerometer measurements. This is modelled in the Kalman filter as additive Gaussian white noise.

Equations (10.20) and (10.22) may be combined and expressed in state space form as:

$$\boldsymbol{\delta}\dot{\mathbf{x}} = \mathbf{F}\boldsymbol{\delta}\mathbf{x} + \mathbf{G}\mathbf{w} \tag{10.23}$$

where $\boldsymbol{\delta}\mathbf{x}$ is the error state vector, $\mathbf{F}$ is the system error matrix, $\mathbf{G}$ is the noise input matrix and $\mathbf{w}$ is the system noise which represents the instrument noise together with any unmodelled biases. The error state vector may be expressed in component form as:

$$\boldsymbol{\delta}\mathbf{x} = [\delta\alpha \quad \delta\beta \quad \delta\gamma \quad \delta v_{\mathrm{N}} \quad \delta v_{\mathrm{E}}]^{\mathrm{T}} \tag{10.24}$$

where $\delta\alpha$, $\delta\beta$, $\delta\gamma$ are the components of the vector $\boldsymbol{\Psi}$, the attitude errors; and δv_{N}, δv_{E} are the north and east velocity errors, respectively.

The error equation may be expressed in full as follows:

$$\begin{pmatrix} \delta\dot{\alpha} \\ \delta\dot{\beta} \\ \delta\dot{\gamma} \\ \delta\dot{v}_{\mathrm{N}} \\ \delta\dot{v}_{\mathrm{E}} \end{pmatrix} = \begin{pmatrix} 0 & \omega_{\mathrm{D}} & -\omega_{\mathrm{E}} & 0 & 0 \\ -\omega_{\mathrm{D}} & 0 & \omega_{\mathrm{N}} & 0 & 0 \\ \omega_{\mathrm{E}} & -\omega_{\mathrm{N}} & 0 & 0 & 0 \\ 0 & -f_{\mathrm{D}} & f_{\mathrm{E}} & 0 & 0 \\ f_{\mathrm{D}} & 0 & -f_{\mathrm{N}} & 0 & 0 \end{pmatrix} \begin{pmatrix} \delta\alpha \\ \delta\beta \\ \delta\gamma \\ \delta v_{\mathrm{N}} \\ \delta v_{\mathrm{E}} \end{pmatrix}$$
$$+ \begin{pmatrix} -c_{11} & -c_{12} & -c_{13} & 0 & 0 & 0 \\ -c_{21} & -c_{22} & -c_{23} & 0 & 0 & 0 \\ -c_{31} & -c_{32} & -c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{11} & c_{12} & c_{13} \\ 0 & 0 & 0 & c_{21} & c_{22} & c_{23} \end{pmatrix} \begin{pmatrix} w_{\mathrm{g}x} \\ w_{\mathrm{g}y} \\ w_{\mathrm{g}z} \\ w_{\mathrm{a}x} \\ w_{\mathrm{a}y} \\ w_{\mathrm{a}z} \end{pmatrix} \tag{10.25}$$

where

$\omega_{\mathrm{N}} = \Omega \cos L + v_{\mathrm{E}}/(R_0 + h)$
$\omega_{\mathrm{E}} = -v_{\mathrm{N}}/(R_0 + h)$
$\omega_{\mathrm{D}} = -\Omega \sin L - v_{\mathrm{E}} \tan L/(R_0 + h)$
Ω = Earth's rate
L = latitude
R_0 = radius of the Earth
h = aircraft altitude
$f_{\mathrm{N}}, f_{\mathrm{E}}, f_{\mathrm{D}}$ = north, east and vertical components of vehicle acceleration, respectively
$c_{11}, c_{12}, \ldots$ = direction cosine elements of the matrix $\mathbf{C}_{\mathrm{b}}^{\mathrm{n}}$
$w_{\mathrm{g}x}, w_{\mathrm{g}y}, w_{\mathrm{g}z}$ = gyroscope noise components
$w_{\mathrm{a}x}, w_{\mathrm{a}y}, w_{\mathrm{a}z}$ = accelerometer noise components.

It can be seen from the system error eqn. (10.22) that an acceleration of the aircraft in the north or east direction is required to cause the azimuthal misalignment ($\delta\gamma$) to propagate as a velocity error.

The error model may be augmented by modelling the gyroscope and accelerometer errors explicitly. For example, additional states may be included to represent the fixed biases in the sensor measurements.

To enable the Kalman filter to be mechanised in discrete form, the system error model is converted to a difference equation by integrating between successive measurement instants to give:

$$\boldsymbol{\delta}\mathbf{x}_{k+1} = \boldsymbol{\Phi}_k \boldsymbol{\delta}\mathbf{x}_k + \mathbf{w}_k \tag{10.26}$$

where $\boldsymbol{\Phi}_k = \exp[\mathbf{F}_k(t_{k+1} - t_k)]$, the system transition matrix between time t_k and t_{k+1} and $\mathbf{w}_k$ is a zero mean white noise sequence.

The measurement equations

The measurements of north and east velocity provided by the aircraft's navigation system constitute the Kalman filter measurements ($\tilde{\mathbf{z}}$):

$$\tilde{\mathbf{z}} = \begin{pmatrix} \tilde{v}_\mathrm{N} \\ \tilde{v}_\mathrm{E} \end{pmatrix} \tag{10.27}$$

Estimates of these measurements ($\hat{\mathbf{z}}$) are obtained from the aligning system:

$$\hat{\mathbf{z}} = \begin{pmatrix} \hat{v}_\mathrm{N} \\ \hat{v}_\mathrm{E} \end{pmatrix} \tag{10.28}$$

Where the reference and aligning systems are installed some distance apart on the aircraft, it will be necessary to compensate for the rotation-induced velocity components, $\mathbf{v}_\mathrm{r}$, the lever-arm motion. Such corrections are calculated using measurements of the aircraft's turn rate ($\boldsymbol{\omega}_\mathrm{a}$) and knowledge of the physical separation between the two systems ($\mathbf{r}$) using $\mathbf{v}_\mathrm{r} = \boldsymbol{\omega}_\mathrm{a} \times \mathbf{r}$ resolved in the reference frame. Measurements of $\boldsymbol{\omega}_\mathrm{a}$ may be provided either by the aircraft's navigation system or by the aligning system with sufficient accuracy.

The velocity measurements are compared at each measurement update to generate the filter measurement differences or innovations, denoted as $\boldsymbol{\delta}\mathbf{z}$, where:

$$\boldsymbol{\delta}\mathbf{z} = \begin{pmatrix} \tilde{v}_\mathrm{N} & -\hat{v}_\mathrm{N} \\ \tilde{v}_\mathrm{E} & -\hat{v}_\mathrm{E} \end{pmatrix} = \begin{pmatrix} -\delta v_\mathrm{N} \\ -\delta v_\mathrm{E} \end{pmatrix} \tag{10.29}$$

The measurement differences at time $t_k(\boldsymbol{\delta}\mathbf{z}_k)$ may be expressed in terms of the error states ($\boldsymbol{\delta}\mathbf{x}_k$) as follows:

$$\boldsymbol{\delta}\mathbf{z}_k = \mathbf{H}_k \boldsymbol{\delta}\mathbf{x}_k + \mathbf{v}_k \tag{10.30}$$

where $\mathbf{H}_k$ is the Kalman filter measurement matrix which takes the following form:

$$\mathbf{H}_k = \begin{pmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \tag{10.31}$$

and $\mathbf{v}_k$ is the measurement noise vector. This represents the noise on the reference measurements and model-mismatch introduced through aircraft flexure and lever-arm motion.

The Kalman filter

In eqns. (10.23) and (10.30), we have the necessary system and measurement equations with which to construct a Kalman filter. The form of the filter equations are given in Appendix A.

The filter provides estimates of the attitude errors and the north and east velocity errors. These estimates are used to correct the aligning system estimates of attitude and velocity after each measurement update. Where instrument bias states are included in the error model, the bias estimates so generated may be used to correct the sensor outputs as part of the alignment process. A block diagram representation of the alignment scheme is given in Figure 10.8.

Whilst it is often recommended that the aircraft should perform a well-defined manoeuvre to aid the alignment process, such as the weave trajectory illustrated in Figure 10.9, analysis of the problem has shown that an alignment can often be achieved in the presence of relatively small perturbations, as would be experienced normally during flight.

Example results

Some simulation results which illustrate the alignment that may be achieved using velocity matching are given in Figure 10.10. The results show the reduction in the alignment error of an airborne navigation system, over a period of 100 s, as the aircraft executes a weave manoeuvre, and have been obtained using an filter formulation similar to that described above, but with the addition of instrument bias states. These results were obtained using a typical aircraft quality system, capable of navigating to an accuracy of 1 nautical mile per hour, to provide the reference measurements. The aligning system was of sub-inertial quality incorporating gyroscopes and accelerometers with 1σ biases of 10°/h and 2 milli-g, respectively.

The figure shows the reduction in the standard deviation of the yaw error as a function of time. The roll and pitch errors, which are not shown here, converge very

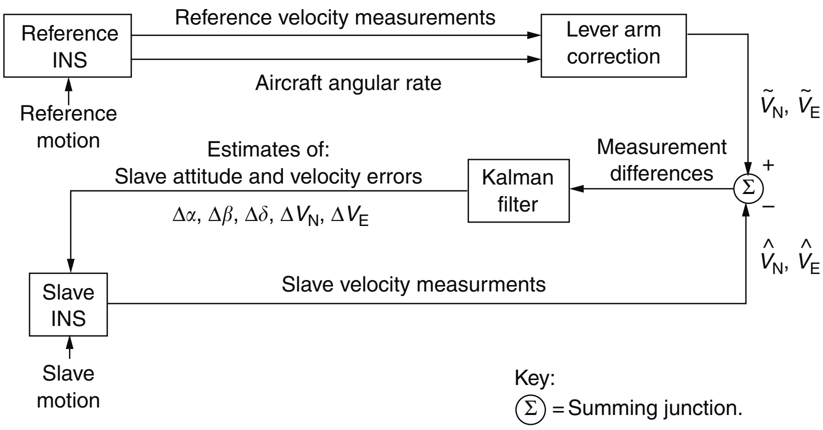

Figure 10.8 Velocity matching alignment scheme

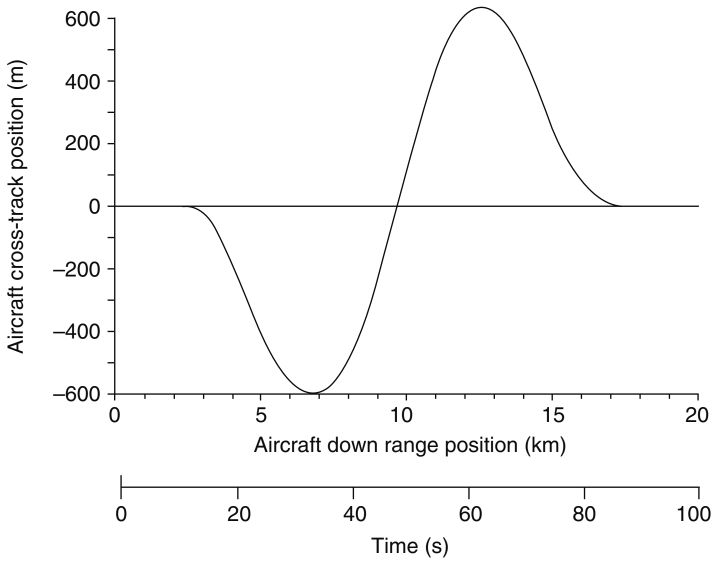

Figure 10.9 Aircraft alignment/calibration manoeuvre

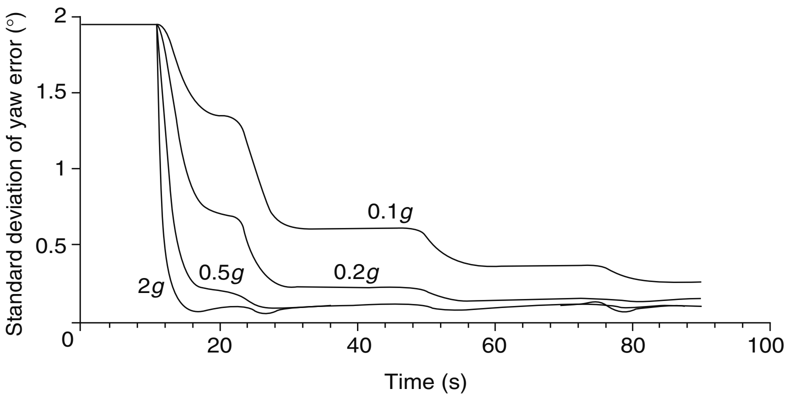

Figure 10.10 Alignment by velocity matching in the presence of an aircraft weave manoeuvre

rapidly as the system effectively aligns itself to the local gravity vector. The accuracy of alignment in level (tilt error) is limited by any residual bias in the accelerometer measurements. In the case shown here, the accelerometer bias is 2 milli-*g*, which results in tilt errors of approximately 0.1°. The yaw alignment error does not begin to converge until the aircraft commences its manoeuvre, since it only propagates as a velocity error and therefore only becomes observable when the aircraft manoeuvres.

The effects of the manoeuvres are clearly shown in the figure. It can be seen that the yaw alignment error falls each time the aircraft starts to change direction.

In the presence of more severe manoeuvres, mean errors also arise which are correlated with the motion of the aircraft. These errors must be summed with the standard deviations shown in the figure to give the full alignment error. The bias terms are principally the result of geometric effects induced as the aircraft banks to turn. Alignment information can only be deduced about axes which are perpendicular to the direction of the applied acceleration, with the result that some redistribution of the alignment errors tends to take place as the aircraft manoeuvres.

10.4.3.4 Position update alignment

An aircraft may be equipped with various sensors or systems capable of providing position fix information which may be used to align an on-board inertial navigation system during flight. Suitable data may be provided by satellite updates [6] or generated through the use of a ground-based tracking radar or a terrain referenced navigation system of the type discussed later in Chapter 13.

As described earlier, position errors will propagate in an inertial navigation system as a result of alignment inaccuracies. By comparing the external position fixes with the estimates of position generated by the aligning navigation system, estimates of the position errors are obtained. Based on a model of the errors in the aligning system it is possible to deduce the alignment errors from these differences in position. A block diagram of such a scheme is given in Figure 10.11.

This method of alignment is precisely equivalent to the inertial aiding process described in Chapter 13. In the context of integrated navigation systems, or aided inertial navigation systems, the external measurements are assumed to be available throughout all or much of the period for which the navigation system is required to navigate. In the context of pre-flight alignment, it refers to the use of the external measurement data purely to carry out an alignment prior to a period of navigation.

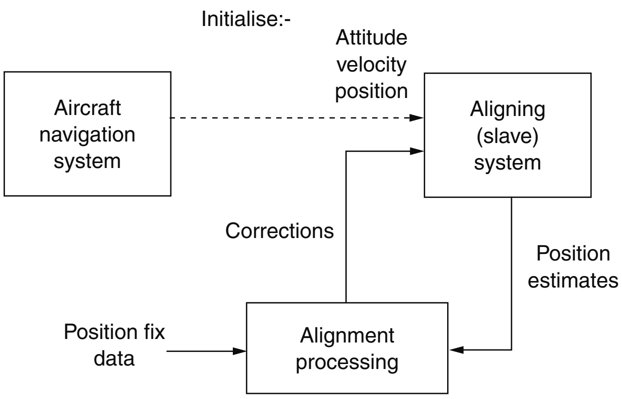

Figure 10.11 Position update alignment scheme

Since the principles of the method are as described in Chapter 13, no further discussion of this topic appears in this chapter.

10.4.3.5 Attitude matching

Recent work has shown that the use of attitude matching, as well as velocity matching, increases the observability of the INS attitude errors, enabling a more accurate alignment to be obtained, or the same accuracy to be obtained with a shorter alignment time or using less manoeuvring of the aircraft. Most importantly, attitude and velocity matching enables an alignment to take place in the presence of a wing rock manoeuvre alone. This is in contrast to velocity matching only which generally requires some heading change manoeuvre, and therefore imposes tactical constraints on the pilot. The attitude difference between the aligning and reference INS is the sum of the attitude error of the aligning INS and the physical relative orientation of the two INS. To separate the two, the Kalman filter must also estimate relative orientation. Adding attitude matching was first proposed by Kain and Cloutier [7]. Flight trials of this technique on a fast jet have been conducted by Graham *et al.* [8] and at QinetiQ, Farnborough [9].

Attitude matching was originally proposed for helicopters where the lever-arm between the reference and aligning INS is relatively rigid. For aircraft where the weapon is mounted on a wing pylon, the flexure environment is more severe. Lever-arm vibration effects can be averaged out by selecting suitably low gains in the Kalman filter. However a more serious problem is presented by the flexure of the wings and pylons in response to aircraft manoeuvre. This can seriously disrupt the performance of transfer alignment using attitude matching. The solution is to introduce additional Kalman filter states that model the variation of the relative orientation with the forces on the wing and to increase the assumed measurement noise in the Kalman filter as a function of the departure of the forces on the wing from their steady state values.

Transfer alignment performance is enhanced by estimating inertial instrument errors as well as velocity and attitude. Estimating accelerometer and gyroscope biases has a huge effect on performance. Further improvements can be attained for some types of IMU by separating the biases into static and dynamic (Markov) states and by estimating scale-factor and cross-coupling errors for both accelerometers and gyros.

The best navigation performance that a transfer aligned INS can attain is that of the reference. Thus, if the aircraft contains an integrated INS–GPS navigation system, this will generally provide a more accurate reference than a pure INS. However, when GPS signals are suddenly re-acquired after a period of jamming (e.g. if the jammer is destroyed) the transient in the aircraft velocity solution as GPS corrects the inertial drift can disrupt the transfer alignment process. The crude solution is to use pure INS as the main transfer alignment reference and just use the integrated solution to correct the weapon position at launch. However, this discards the GPS calibration of the aircraft INS velocity and attitude. Thus, it is better to use the INS–GPS solution as the reference and add a transient handling algorithm.

The best approach to transient handling is to detect transients directly, either by comparing the integrated and pure INS solutions or by taking correction information from the aircraft navigation filter. In this case, the transient is applied to the missile velocity solution outside the transfer alignment Kalman filter to keep it in step. Where this cannot be done, the transfer alignment algorithm must monitor the measurement residuals for the effects of transients and, if it finds one, selectively increase the error covariance, to make the velocity error estimates more receptive to the corrected aircraft solution.

10.5 Alignment at sea

10.5.1 Introduction

A modern warship contains a wide variety of sensors and weapon systems. In order that the ship can deploy the forces at its disposal and use them in an effective manner, all such equipment must operate in harmony. For example, information about an attacking missile or aircraft derived from a sensor at one location must be in a form that can be used to direct or control a weapon system at a different remote location.

10.5.2 Sources of error

It is common practice to set up a series of datum levels and training marks at strategic locations around the ship to which all equipment is referenced or harmonised when it is installed on the ship. In this way, it is hoped to ensure that all equipment will operate in a common frame of reference. It has long been suspected that whilst the accuracy to which equipment is harmonised during the construction of the ship is very high, the accuracy of this harmonisation degrades when the ship goes to sea. This view has been reinforced by observations of ships at sea and the results of ship trials which have attempted to measure the amount by which ships bend or flex in different sea conditions. Such errors may be categorised as follows:

Long-term deformations occurring through the action of ageing and the effects of solar heating. A gradual movement of the structure takes place as the ship ages and as the load state changes. It has also been observed that significant bending of the ship structure can occur under the action of solar heating. Angular variations of the order of 1° are believed to take place over the period of a day as the sun moves around the vessel.

Ship flexure can occur in heavy seas as the ship moves in response to the motion of the waves, the magnitude of the angular displacement between any two locations becoming larger as the separation increases. Attempts to measure the amount by which ships flex when at sea have revealed significant angular displacements at typical ship motion frequencies of 0.1–0.3 Hz, the dominant flexure motion being the twisting of the hull about the roll axis of the vessel. The magnitude of ship flexure is a function of sea state and the direction in which the waves are approaching the vessel. Further transient distortion may occur as the ship manoeuvres, or through the action of the stabilisers.

Other abrupt changes which are expected to arise from underwater shock, induced for instance by a depth charge, and as a result of slamming in heavy seas, where the bows leave the water and impact on re-entry.

In addition, battle damage will introduce potentially very large distortions of a ship's structure, probably rendering some weapon systems ineffective unless a static reharmonisation takes place.

10.5.3 Shipboard alignment methods

To overcome the problems outlined in the previous section, it is necessary to devise means by which the harmonisation of the various shipboard systems can be maintained under all operational conditions. Whilst an accurate reference is provided on naval ships by the ship's attitude and heading reference system (AHRS) or even more precisely by a ship's inertial navigation system (SINS), the accuracy with which that reference may be transferred about the ship is limited by bending and flexure of the ship. For this reason, other means are sought for the alignment of equipment on-board ships.

10.5.3.1 Shipboard transfer alignment methods

Assuming a master reference can be maintained accurately, slave systems may be aligned to that reference. There are various methods which may be adopted to achieve this end. The simplest technique is to transfer data – attitude, velocity and position– directly from the master system to the slave using the one-shot alignment scheme described above for airborne alignment. However, as with airborne alignment, any physical misalignments resulting from ship flexure, for example, will contribute directly to the errors in the aligning system if this approach is adopted.

One possible method of overcoming this limitation on-board a ship is to use an optical harmonisation scheme to determine the relative orientation of the master reference of the launch platform and a missile system directly. An auto-collimator, fixed in one co-ordinate reference frame, may be used to determine the rotation of a reflector which is attached to the second reference frame. Although such techniques have been used in some applications, they are not generally feasible because of the difficulty of maintaining line-of-sight contact between the two locations which could be some considerable distance apart. For example, a missile silo in a ship may be installed 50 m, or more, away from the ship's inertial reference system.

Alternatively, alignment may be achieved on board a ship by comparing inertial measurements generated by the aligning system with similar measurements provided by a reference unit [10, 11]. The velocity matching scheme described in Section 10.4 for in-flight alignment is of limited use for shipboard applications since it is dependent on a manoeuvre of the vehicle, particularly if an alignment is to take place within a short period of time. In many circumstances this may be totally impractical. Studies of shipboard alignment methods have suggested that the use of velocity and pitch rate matching offers a possible solution [11]. Such a scheme is discussed in more detail in the following section.

10.5.3.2 Shipboard inertial measurement matching

In this section, the scope for achieving an alignment at sea using velocity and angular rate matching is discussed. The application of velocity matching alone is of limited use for shipboard alignment because ships are clearly unable to manoeuvre in the way that aircraft can to aid the alignment process. However, velocity matching may be used to achieve a level alignment, since errors in the knowledge of the local vertical will cause the measurements of specific force needed to overcome gravity to be resolved incorrectly and to propagate as apparent components of north and east velocity.

On-board a ship, an alignment in azimuth may be achieved within a relatively short period of time by comparing angular rate measurements, provided the ship exhibits some motion in pitch or roll. The measurements may be processed using a Kalman filter based on an error model of the aligning system, as described in the context of in-flight alignment in Section 10.4. The form of the measurement equation is described below.

The measurements of turn rate provided by the reference and aligning systems are assumed to be generated in local co-ordinate frames denoted a and b, respectively. The rates sensed by a triad of strapdown gyroscopes mounted at each location with their sensitive axes aligned with these reference frames may be expressed as $\boldsymbol{\omega}_{\mathrm{ia}}^{\mathrm{a}}$ and $\boldsymbol{\omega}_{\mathrm{ib}}^{\mathrm{b}}$ in line with the nomenclature used in Chapter 3. The measurements provided by the gyroscopes in the reference and aligning systems are resolved into a common reference frame, the a-frame, for instance, before comparison takes place.

Hence, the reference measurements may be expressed as:

$$\tilde{\mathbf{z}} = \boldsymbol{\omega}_{\mathrm{ia}}^{\mathrm{a}} \tag{10.32}$$

assuming errors in the measurements to be negligible. The estimates of these measurements generated by the aligning system are denoted by the $\wedge$ notation.

$$\hat{\mathbf{z}} = \hat{\mathbf{C}}_{\mathrm{b}}^{\mathrm{a}} \hat{\boldsymbol{\omega}}_{\mathrm{ib}}^{\mathrm{b}} \tag{10.33}$$

The gyroscope outputs ($\hat{\boldsymbol{\omega}}_{\mathrm{ib}}^{\mathrm{b}}$) may be written as the sum of the true rate ($\boldsymbol{\omega}_{\mathrm{ib}}^{\mathrm{b}}$) and the error in the measurement ($\delta\boldsymbol{\omega}_{\mathrm{ib}}^{\mathrm{b}}$) whilst the estimated direction cosine matrix may be expressed as the product of a skew symmetric error matrix, $[\mathbf{I} - \boldsymbol{\Psi}]$, and the true matrix $\mathbf{C}_{\mathrm{b}}^{\mathrm{a}}$ to give:

$$\hat{\mathbf{z}} = [\mathbf{I} - \boldsymbol{\Psi}]\mathbf{C}_{\mathrm{b}}^{\mathrm{a}}[\boldsymbol{\omega}_{\mathrm{ib}}^{\mathrm{b}} + \delta\boldsymbol{\omega}_{\mathrm{ib}}^{\mathrm{b}}]$$

Expanding the right-hand side of this equation, writing $\boldsymbol{\Psi} = [\boldsymbol{\psi}\times]$ and ignoring error product terms gives:

$$\begin{aligned}\hat{\mathbf{z}} &= \mathbf{C}_{\mathrm{b}}^{\mathrm{a}}\boldsymbol{\omega}_{\mathrm{ib}}^{\mathrm{b}} - \boldsymbol{\psi} \times \mathbf{C}_{\mathrm{b}}^{\mathrm{a}}\boldsymbol{\omega}_{\mathrm{ib}}^{\mathrm{b}} + \mathbf{C}_{\mathrm{b}}^{\mathrm{a}}\delta\boldsymbol{\omega}_{\mathrm{ib}}^{\mathrm{b}} \\ &= \boldsymbol{\omega}_{\mathrm{ib}}^{\mathrm{a}} + \boldsymbol{\omega}_{\mathrm{ib}}^{\mathrm{a}} \times \boldsymbol{\psi} + \mathbf{C}_{\mathrm{b}}^{\mathrm{a}}\delta\boldsymbol{\omega}_{\mathrm{ib}}^{\mathrm{b}}\end{aligned} \tag{10.34}$$

The turn rate of the aligning system may be expressed as the sum of the turn rate sensed by the reference system and any ship flexure which may be present ($\boldsymbol{\omega}_{\mathrm{f}}$). Hence, eqn. (10.34) may be rewritten as follows:

$$\hat{\mathbf{z}} = \boldsymbol{\omega}_{\mathrm{ia}}^{\mathrm{a}} + \boldsymbol{\omega}_{\mathrm{f}} + \boldsymbol{\omega}_{\mathrm{ib}}^{\mathrm{a}} \times \boldsymbol{\psi} + \mathbf{C}_{\mathrm{b}}^{\mathrm{a}}\delta\boldsymbol{\omega}_{\mathrm{ib}}^{\mathrm{b}} \tag{10.35}$$

The measurement differences may then be written as:

$$\begin{aligned}\boldsymbol{\delta}\mathbf{z} &= \tilde{\mathbf{z}} - \hat{\mathbf{z}} \\ &= -\boldsymbol{\omega}_{\mathrm{ib}}^{\mathrm{a}} \times \boldsymbol{\psi} - \mathbf{C}_{\mathrm{b}}^{\mathrm{a}}\boldsymbol{\delta\omega}_{\mathrm{ib}}^{\mathrm{b}} - \boldsymbol{\omega}_{\mathrm{f}}\end{aligned} \tag{10.36}$$

The measurement differences ($\boldsymbol{\delta}\mathbf{z}_k$) at time t_k may be expressed in terms of the error states ($\boldsymbol{\delta}\mathbf{x}_k$) as follows:

$$\boldsymbol{\delta}\mathbf{z}_k = \mathbf{H}_k\boldsymbol{\delta}\mathbf{x}_k + \mathbf{v}_k \tag{10.37}$$

where $\mathbf{H}_k$ is the Kalman filter measurement matrix which takes the following form:

$$\mathbf{H}_k = \begin{pmatrix} 0 & \omega_Z & -\omega_Y & 0 & 0 \\ -\omega_Z & 0 & \omega_X & 0 & 0 \\ \omega_Y & -\omega_X & 0 & 0 & 0 \end{pmatrix} \tag{10.38}$$

where ω_X, ω_Y and ω_Z are the components of the vector $\boldsymbol{\omega}_{\mathrm{ib}}^{\mathrm{a}}$ and $\mathbf{v}_k$ is the measurement noise vector. This represents the noise on the measurements and model-mismatch introduced through ship flexure.

A Kalman filter may now be constructed using the measurement eqn. (10.37) and a system equation of the form described earlier, Section 10.4.3.3, eqn. (10.23). A block diagram of the resulting alignment scheme is given in Figure 10.12.

Example result

The simulation result shown in Figure 10.13 illustrates the accuracy of alignment which may be achieved using a combination of velocity and angular rate matching. The results show the convergence of the azimuthal alignment error in calm, moderate and rough sea conditions where the waves are approaching the ship from the side.

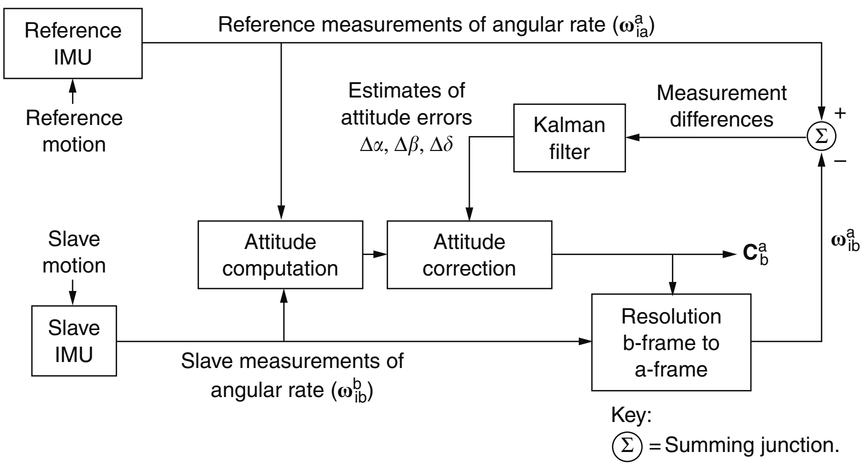

Figure 10.12 Angular rate matching alignment scheme

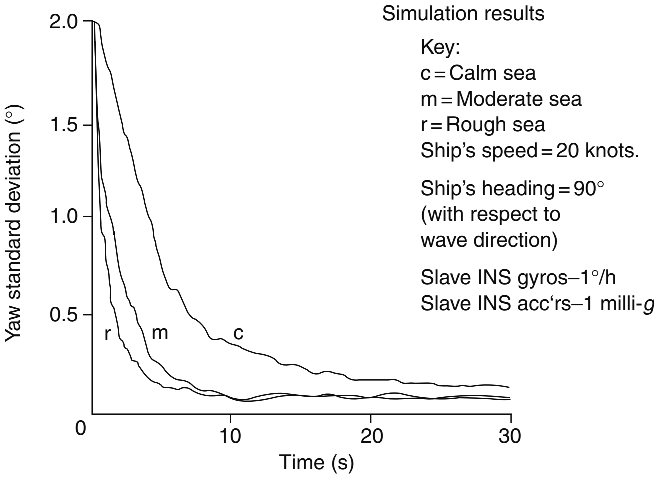

Figure 10.13 Illustration of measurement matching at sea

These results were obtained assuming no knowledge of the ship's flexure characteristics. However, the measurements of velocity were compensated for relative motion of the reference and aligning systems caused by the rotation of the ship. The aligning system contained medium grade inertial sensors with accelerometer biases of 1 milli-g and gyroscope biases of 1°/h; a higher quality reference system was used. The Kalman filter used here was found to be robust in that it is able to cope with initial alignment errors of 10° or more.

The effects of ship flexure

Whilst it is possible in theory to model the ship's flexure explicitly in the Kalman filter and so derive estimates of the flexure rates, a sufficiently precise model is unlikely to be available in practice. Besides, this will result in a 'highly tuned' filter which will be very sensitive to parametric variations. For these reasons, a sub-optimal Kalman filter may be used in which the flexure is represented as a noise process, as described above. The way in which ship flexure limits the accuracy of alignment which can be achieved when using a filter of this type is demonstrated by the simplified analysis which follows.

Consider the two axis sets shown in Figure 10.14 which correspond to the orientations of the reference and aligning systems at two locations remote from each other on a ship. The reference frame is taken to be aligned perfectly with the roll, pitch and yaw axes of the ship whilst the aligning system, denoted here as the slave system, is misaligned in yaw by an angle $\delta\psi$.

In Figure 10.14, O_aXY denotes reference axes at reference system origin; O_bXY denotes a parallel reference axes at the slave system origin and O_bxy denotes the slave system axes to be brought into alignment with O_bXY.

The angular rates p and q sensed about the reference axes are the roll and pitch rates of the vessel, respectively. The slave system senses rates $p + \delta p$ and $q + \delta q$

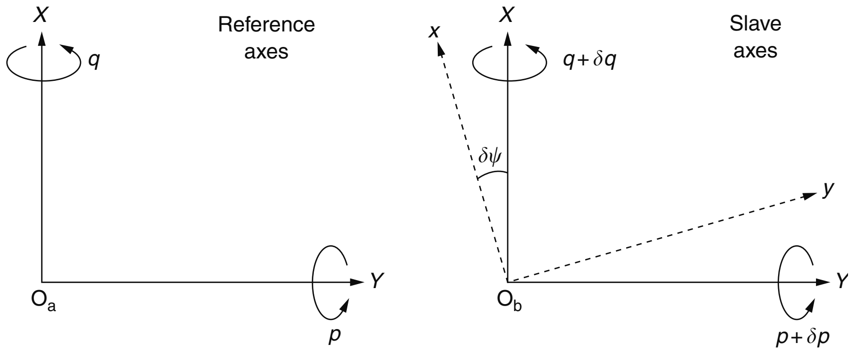

Figure 10.14 Illustration of the effects of ship flexure on axis alignment

resolved in slave system axes, where δp and δq represent the relative angular rates between the two systems, the rates at which the ship is bending or flexing.

Consider first the mechanism by which alignment occurs in the absence of flexure. Using pitch rate matching, the rate measured by the reference system, q, is compared with the slave system rate, $q\cos\delta\psi - p\sin\delta\psi$, to yield a measurement difference δz, where:

$$\delta z = q(1 - \cos\delta\psi) + p\sin\delta\psi \tag{10.39}$$

It can be seen from the above equation that δz becomes zero when the misalignment is zero. Hence, by adjusting $\delta\psi$ in order to null this measurement difference, it is possible to align the slave system perfectly in the absence of ship flexure.

In the presence of ship flexure, additional turn rates δp and δq are present at the slave system and the rate sensed about the nominal pitch axis of the slave system becomes $(q + \delta q)\cos\psi - (p + \delta p)\sin\psi$. The measurement difference is now:

$$\delta z = q(1 - \cos\delta\psi) + p\sin\delta\psi - \delta q\cos\delta\psi + \delta p\sin\delta\psi \tag{10.40}$$

which may be expressed to first order in $\delta\psi$ as:

$$\delta z = (p + \delta p)\delta\psi - \delta q \tag{10.41}$$

In this case, the measurement difference settles to zero when:

$$\delta\psi = \frac{\delta q}{(p + \delta p)} \tag{10.42}$$

It is clear from this result that the magnitude of the residual yaw misalignment will reduce as the roll rate of the ship becomes larger, or as the flexure about the measurement axis, pitch in this case, becomes smaller. By a similar argument, it can be shown that the accuracy of the estimate of yaw error obtained using roll rate matching will be limited by the relative magnitude of the roll flexure and the pitch rate of the vessel. Since flexure about the roll axis is believed to be larger than the pitch rate flexure in general, and ships tend to roll more rapidly than they pitch, pitch rate matching is the preferred option.

Figure 10.15 shows the azimuthal alignment accuracy achieved as the ratio of pitch rate flexure to roll rate is varied. In line with theoretical expectations discussed here, the accuracy of alignment is shown to improve as this ratio becomes smaller.

10.5.3.3 Shipboard alignment using position fixes

Accurate harmonisation between different items of equipment on-board a ship may be achieved using inertial navigation systems installed alongside each item, or system, to maintain a common reference frame at each location. Such a scheme is shown in Figure 10.16.

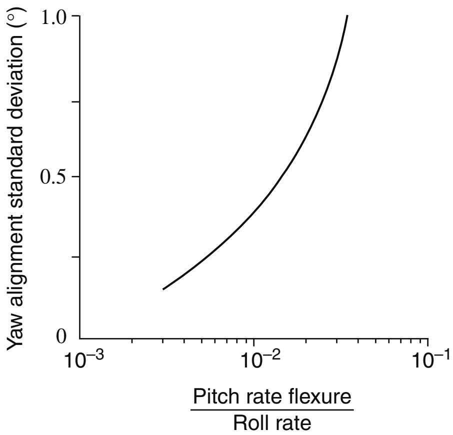

Figure 10.15 Azimuth alignment accuracy as a function of the ratio pitch rate flexure to roll rate

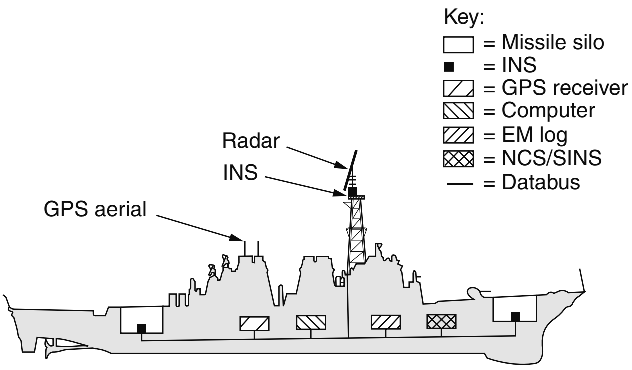

Figure 10.16 Shipboard harmonisation scheme

The reference may be maintained at each location by using accurate position fixes, provided by satellite updates for instance. It is envisaged that each system could be equipped with a GPS satellite receiver and antenna to facilitate its alignment to the local geographic frame. Alternatively, with appropriate filtering and lever-arm corrections, a single GPS receiver could feed all of the inertial systems on the ship with positional data. It is noted that the GPS receiver gives the location of the phase centre of the antenna which is likely to be located at the top of a mast.

The alignment of each inertial system may be accomplished independently of ship motion, although the speed of convergence is greatly increased in the presence of the ship's manoeuvres. This technique would enable the accurate alignment of each system to be achieved, largely irrespective of any relative motion between the different locations resulting from bending of the ship's structure. Clearly, this approach is dependent on the continuing availability of satellite signals. In the event of loss of such transmissions, the period of time for which alignment can subsequently be maintained is dependent on the quality and characteristics of the sensors in each inertial unit.

References

1 BRITTING, K.R.: 'Inertial navigation system analysis' (John Wiley and Sons, 1971)
2 DEYST, J.J., and SUTHERLAND, A.A.: 'Strapdown inertial system alignment using statistical filters: a simplified formulation', *AIAA Journal*, 1973, **11** (4)
3 HARRIS, R.A., and WAKEFIELD, C.D.: 'Co-ordinate alignment for elastic bodies', *NAECON* 1977
4 SCHULTZ, R.L., and KEYS, C.L.: 'Airborne IRP alignment using acceleration and angular rate matching'. Proceedings *Joint automatic control conference*, June 1973
5 BAR-ITZHACK, I.Y., and PORAT, B.: 'Azimuth observability enhancement during inertial navigation system in-flight alignment', *Journal of Guidance and Control*, 1980, **3** (4)
6 TAFEL, R.W., and KRASNJANSKI, D.: 'Rapid alignment of aircraft strapdown inertial navigation systems using Navstar GPS'. AGARD conference proceedings on *Precision positioning and guidance systems*, 1980
7 KAIN, J.E., and CLOUTIER, J.R.: 'Rapid transfer alignment for tactical weapon applications'. Proceedings of *AIAA Guidance, Navigation and Control Conference*, 1989
8 GRAHAM, W.R., SHORTELLE, K.J., and RABOURN, C.: 'Rapid alignment prototype (RAP) flight test demonstration'. Proceedings of the *Institute of Navigation National Technical Meeting*, 1998
9 GROVES, P.D.: 'Optimising the transfer alignment of weapon INS', *Journal of Navigation*, 2003, **56**

10 BROWNE, B.H., and LACKOWSKI, D.H.: 'Estimation of dynamic alignment error in shipboard fire control systems'. Proceedings of IEE conference on *Decision and control*, 1976

11 TITTERTON, D.H., and WESTON, J.L.: 'Dynamic shipboard alignment techniques'. DGON proceedings, *Gyro Technology Symposium*, Stuttgart, 1987